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CRITICAL EXCITATION METHODS FOR IMPORTANT STRUCTURES

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ABSTRACT

The role of critical excitation methods in the design of important structures is discussed extensively. The problem of critical excitation for time-dependent performance indices is treated first. Then the problem of critical excitation for earthquake input energy is discussed. The application of various critical excitation methods to high-rise buildings, base-isolated buildings, nuclear reactor facilities, etc. is presented.

1. INTRODUCTION

There are various buildings in a city. Each building has its own natural period and its original structural properties. When an earthquake occurs, a variety of ground motions are induced in the city. The combination of the building natural period with the predominant period of the induced ground motion may lead to disastrous phenomena in the city. Many past earthquake observations demonstrated such phenomena repeatedly. Once a big earthquake occurs, some building codes are upgraded. However, it is true that this repetition never resolves all the issues and new serious problems occur even recently. In order to overcome this problem, a new paradigm has to be posed. To the author's knowledge, the concept of "critical excitation" [1, 2] and the structural design based upon this concept can become one of such new paradigms [3, 4].

Earthquake inputs are uncertain even with the present knowledge and it does not appear easy to predict forthcoming events precisely both in time-history and frequency contents [5, 6]. For example, recent near-field ground motions (Northridge 1994, Kobe 1995, Turkey 1999 and Chi-Chi, Taiwan 1999) and the Mexico Michoacan motion 1985 have some peculiar characteristics unpredictable before their occurrence. It is also true that the civil, mechanical and aerospace engineering structures are often required to be designed for disturbances including inherent uncertainties due mainly to their "low rate of occurrence". Worst-case analysis combined with proper information based on reliable physical data is expected to play an important role in avoiding difficulties induced by such uncertainties. Approaches based on the concept of "critical excitation" seem to be promising.

The most critical issue in the seismic resistant design is the resonance. One of the promising approaches to this phenomenon is to shift the natural period of the building through seismic control [7] and to add damping in the building. However it is also true that the seismic control is under development and more sufficient time is necessary to respond to uncertain ground motions.

It is believed that earthquake has a bound on its magnitude. In other words, the earthquake energy radiated from the fault has a bound [8]. The problem is to find the most unfavourable ground motion for a building or a group of buildings (see Fig.1). The Fourier spectrum of a ground motion acceleration has been proposed at the rock surface depending on the seismic moment M_0, distance R from the fault, etc. (Fig.2; for example see [9]).

$$\left|A(\omega)\right| = CM_0 S(\omega, \omega_C) P(\omega, \omega_{\max}) \exp(-\omega R / (2\beta Q_\beta)) / R \tag{1}$$

Such spectrum may have uncertainties. One possibility or approach is to specify the acceleration or velocity power and allow the variability of the spectrum (see Fig.3).

The problem of ground motion variability is very important and tough. Code-specified design ground motions are usually constructed by taking into account the knowledge from the past observation and the probabilistic insights. However, uncertainties in the occurrence of earthquakes (or ground motions), the fault rupture mechanisms, the wave propagation mechanisms, the ground properties, etc. cause much difficulty in defining reasonable design ground motions especially for important buildings in which severe damage or collapse has to be avoided absolutely [4, 5, 10-13].

A long-period ground motion has been observed in USA and Japan recently. This type of ground motion is told to cause a large seismic demand to such structures as high-rise buildings, base-isolated buildings, oil tanks, etc, which have long natural periods. This large seismic demand results from the resonance between the long-period ground motion and the long natural period of these constructed facilities.

A significance of critical excitation is supported by its broad perspective. There are two classes of buildings in a city (see Fig.4). One is the important buildings which play an important role during and after disastrous earthquakes. The other one is ordinary buildings. The former one should not have damage during an earthquake and the latter one may be damaged partially especially for critical excitation larger than code-specified design earthquakes. Just as the investigation on limit states of structures plays an important role in the specification of allowable response and performance levels of structures during disturbances, the clarification of critical excitations for a given structure or a group of structures appears to provide structural designers with useful information in determining excitation parameters in a reasonable way. The concept of critical excitation may enable structural designers to make ordinary buildings more seismic-resistant.

In the case where influential active faults are known in the design stage of a structure (especially an important structure), the effects by these active faults should be taken into account in the structural design through the concept of critical excitation. If influential active faults are not necessarily known in advance, virtual or scenario faults with an appropriate *energy* may be defined especially in the design of important and socially influential structures. The combination of worst-case analysis [4, 14, 15] with appropriate specification of energy levels [9] derived from the analysis of various factors, e.g. fault rupture mechanism and earthquake occurrence probability, appears to lead to the construction of a more robust and reliable seismic resistant design method. The appropriate setting of energy levels or information used in the worst-case analysis is important and the research on this subject should be conducted more extensively.

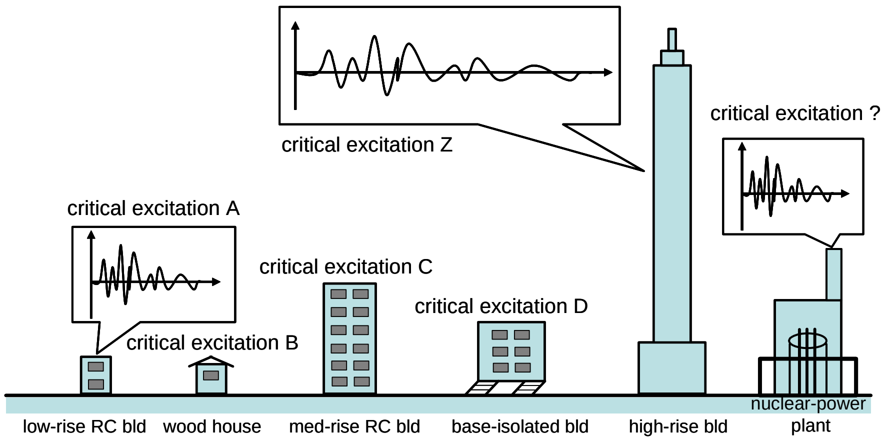


Fig.1 Structure-dependent critical excitation

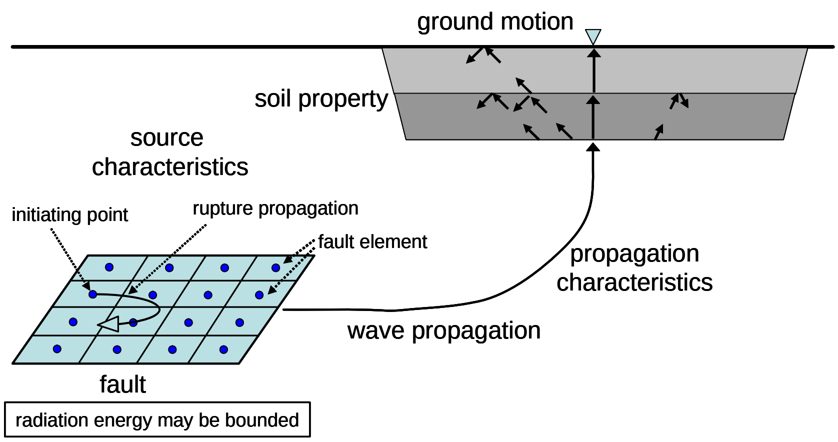


Fig.2 Ground motion affected by source characteristics, propagation characteristics and surface soil properties

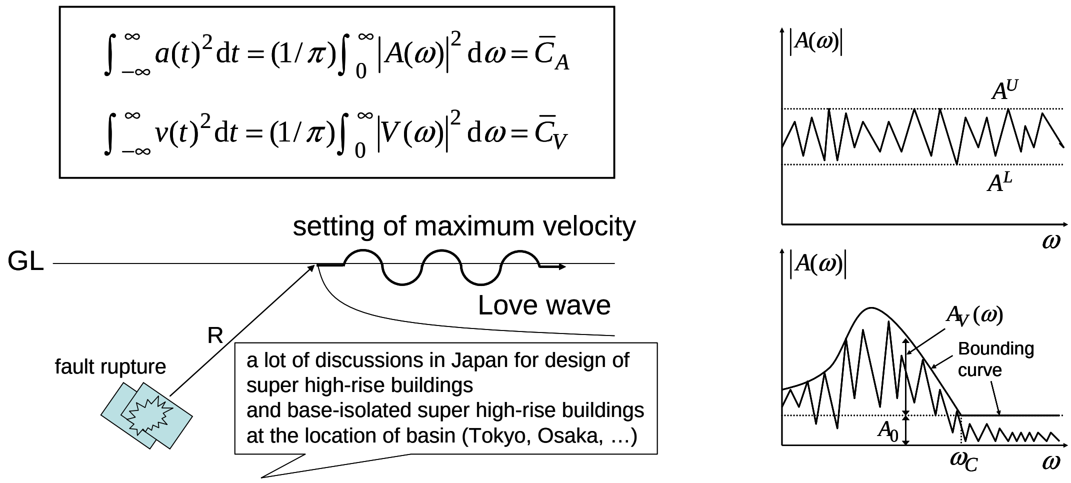


Fig.3 Constraints on acceleration power and velocity power, and bounding of Fourier spectrum of the acceleration input

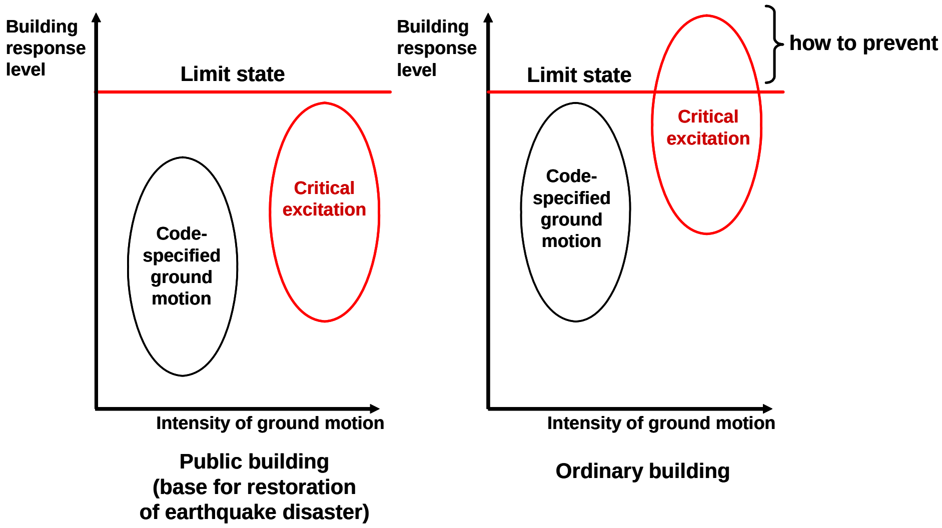


Fig.4 Relation of critical excitation with code-specified ground motion in public and ordinary buildings

2. CRITICAL EXCITATION METHODS FOR TIME-DEPENDENT PERFORMANCE INDICES

In this section, critical excitation methods for stationary and non-stationary random inputs are discussed. It is natural to assume that earthquake ground motions are samples or realizations of a non-stationary random process. Therefore critical excitation methods for non-stationary random inputs may be desirable for constructing and developing realistic earthquake-resistant design methods. However it may also be relevant to develop the critical excitation methods for stationary random inputs as the basis for further development for non-stationary random inputs. First of all, critical excitation methods for stationary random inputs are discussed and some fundamental and important results are derived. These results play a significant role in the critical excitation methods for non-stationary random inputs. Secondly, critical excitation methods for non-stationary random inputs are developed based on the theory for stationary random inputs.

Consider a typical critical excitation problem. The response performance function may be a storey drift [12, 16-21], a floor acceleration [22] or an earthquake energy input rate [23].

The input base acceleration model may be defined by

$$\ddot{u}_g(t) = c(t) w(t) \tag{2}$$

where $c(t)$ is the prescribed deterministic envelope function and $w(t)$ is the stochastic function to be obtained. The power spectral density function of $w(t)$ is denoted by $S_w(\omega)$.

The critical excitation problem may be stated as:

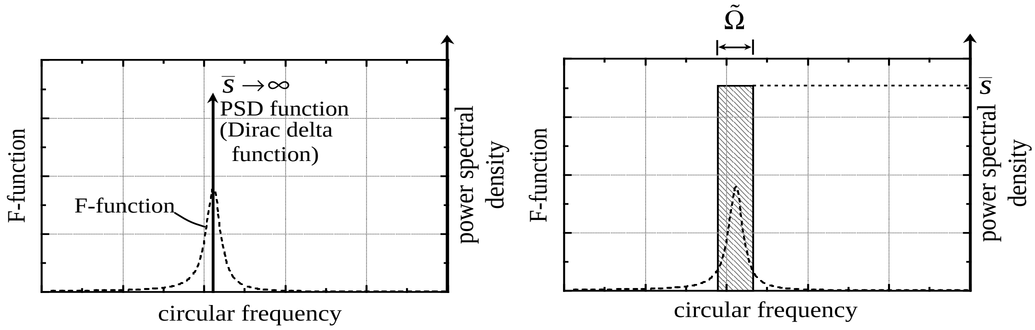
Find $S_w(\omega)$

such that $\max\limits_{S_w(\omega)} \max\limits_{t} \{ f(t; S_w(\omega)) \}$ is achieved

subject to $\int_{-\infty}^{\infty} S_w(\omega) \mathrm{d}\omega \le \overline{S}_w$ $\quad$ ($\overline{S}_w$: given power limit)

and $\sup S_w(\omega) \le \overline{s}$ $\quad$ ($\overline{s}$: given PSD amplitude limit)

where $f = \int_{-\infty}^{\infty} H(t;\omega) S_w(\omega) \mathrm{d}\omega$.



(a) critical excitation for infinite $\bar{s}$ (b) critical excitation for finite $\bar{s}$

Fig.5 Power spectral density function of critical excitation for stationary input

This problem can be solved by using the solution method for stationary problems [16]. A critical excitation method for the corresponding stationary random inputs can be developed by introducing a stochastic response index (mean square response) as the objective function to be maximized. The power and the intensity of the excitations are prescribed so as to be described in the problem just before. Fig.5 shows the schematic diagram of the solution procedure for the corresponding stationary problem. When the restriction on the excitation intensity does not exist, the critical PSD function is turned out to be the Dirac delta function. In the existence of the restriction on the excitation intensity, the critical excitation is reduced to a rectangular PSD function with the maximum intensity limit.

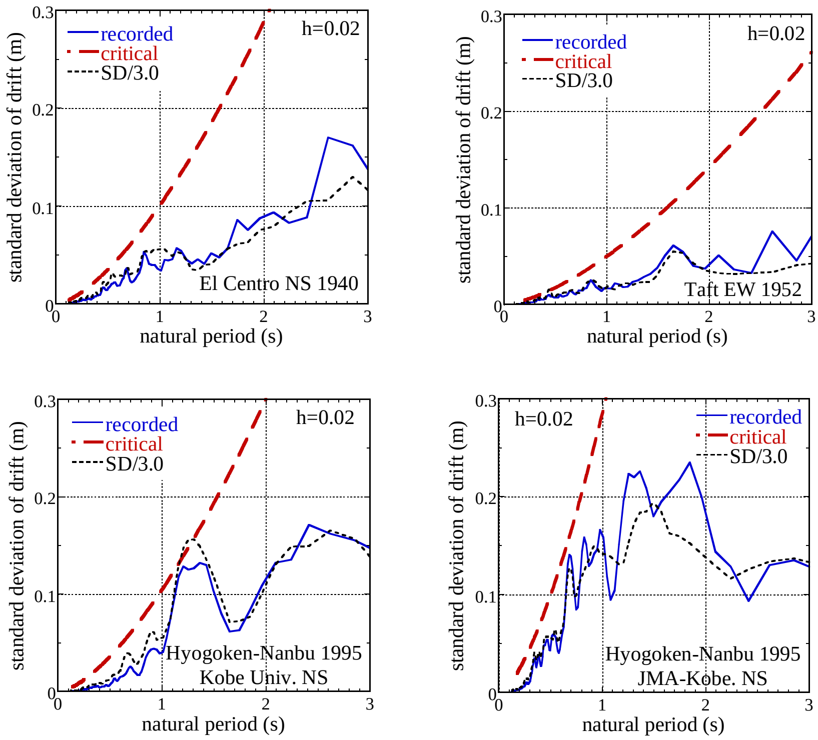


Fig.6 Standard deviation of interstorey drift of an SDOF model (damping ratio=0.02) subjected to recorded earthquakes, that to the present critical excitation and one-third of displacement response spectrum

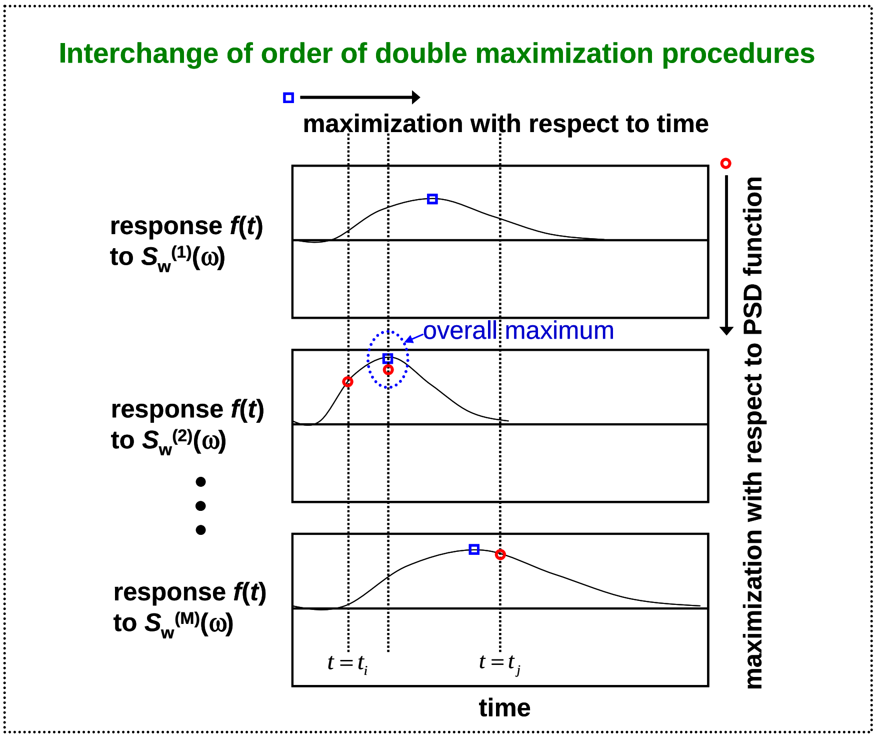


Fig.7 Schematic diagram of the procedure for finding the critical excitation for non-stationary random inputs (order interchange of double maximization procedures)

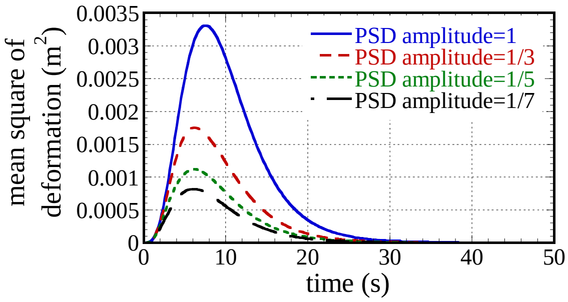


Fig.8 Time-dependent mean-square deformation of an SDOF model for various PSD amplitudes

Fig.6 shows the standard deviation of interstorey drift of a single-degree-of-freedom (SDOF) model subjected to recorded earthquakes, that to the present critical excitation and one-third of displacement response spectrum. The level of conservativeness of the derived critical excitation for stationary random inputs is about 2 or 3 in recorded ground motions without conspicuous predominant frequency. On the other hand, it is close to unity in ground motions with a conspicuous predominant frequency as shown in Fig.6. The resonant characteristic of such ground motion can be represented suitably by the critical excitation.

The key idea for stationary random inputs can be used in SDOF models under non-stationary random inputs which can be described by a uniformly modulated excitation model. The interchange of the order of the double maximization procedures as shown Fig.7 is taken full advantage of with respect to time and to the PSD function. A similar algorithm can also be devised for multi-degree-of-freedom (MDOF) proportionally damped models [17].

Fig.8 shows the time-dependent mean-square deformation of an SDOF model for various PSD amplitudes [17].

3. CRITICAL EXCITATION METHODS FOR EARTHQUAKE INPUT ENERGY

The purpose of this section is to explain a general critical excitation method for a damped linear elastic SDOF system. The input energy to the SDOF system during an earthquake is introduced as a measure of criticality. It is shown that the formulation of the earthquake input energy in the frequency domain is essential for solving the critical excitation problem and deriving a bound on the earthquake input energy for a class of ground motions. The criticality is expressed in terms of degree of concentration of input motion components on the maximum portion of the characteristic function defining the earthquake input energy. It is remarkable that no mathematical programming technique is required in the solution procedure. The constancy of earthquake input energy [24, 25] for various natural periods and damping ratios is discussed from a new point of view based on an original sophisticated mathematical treatment. It is shown that the constancy of earthquake input energy is directly related to the uniformity of 'the Fourier amplitude spectrum' of ground motion acceleration, not the uniformity of the velocity response spectrum as described in [24, 25]. The bounds under acceleration and velocity constraints (time integral of the squared base acceleration and time integral of the squared base velocity) are clarified through numerical examinations for recorded ground motions to be meaningful in the short and intermediate/long natural period ranges, respectively.

Consider an SDOF model of mass *m* as shown in Fig.9. The displacement of the mass relative to the ground is described by *x*. When this model is subjected to the ground motion acceleration $\ddot{u}_g$, the earthquake input energy can be expressed by

$$E_I = \int_0^{t_0} m(\ddot{u}_g + \ddot{x})\dot{u}_g \mathrm{d}t \tag{3}$$

The earthquake input energy can also be expressed in frequency domain by

$$\begin{aligned} E_I / m &= -\int_{-\infty}^{\infty} \dot{x} a \mathrm{d}t = -\int_{-\infty}^{\infty} \left[(1/2\pi)\int_{-\infty}^{\infty} \dot{X} e^{\mathrm{i}\omega t} \mathrm{d}\omega \right] a \mathrm{d}t \\ &= -(1/2\pi)\int_{-\infty}^{\infty} A(-\omega)\left\{ H_V(\omega;\Omega,h) A(\omega) \right\} \mathrm{d}\omega \\ &= \int_0^{\infty} \left| A(\omega) \right|^2 \left\{ -\mathrm{Re}[H_V(\omega;\Omega,h)]/\pi \right\} \mathrm{d}\omega \\ &\equiv \int_0^{\infty} \left| A(\omega) \right|^2 F(\omega) \mathrm{d}\omega \end{aligned} \tag{4}$$

where $H_V(\omega;\Omega,h) = -\mathrm{i}\omega / (\Omega^2 - \omega^2 + 2\mathrm{i}h\Omega\omega)$ is the velocity transfer function.

The energy transfer function $F(\omega)$ characterizing the earthquake input energy to a damped linear elastic SDOF system in the frequency domain has been proved by the Residue Theorem to have an equi-area property, as shown in Fig.10, irrespective of natural period and damping ratio. This property guarantees that, if the Fourier amplitude spectrum of a ground acceleration is uniform, the constancy of the input energy defined by Eq.(4) holds strictly. Otherwise, its constancy is not guaranteed. The constancy of the earthquake input energy defined by Eq.(4) is *directly* related to the constancy of 'the Fourier amplitude spectrum' of the ground acceleration, not the constancy of the velocity response spectrum. Fig.11 shows the schematic diagram of solution procedure for critical excitation problem with acceleration constraints. The energy transfer function for velocity constraint is presented in Fig.12 and the schematic diagram of solution procedure for velocity constraints is shown in Fig.13.

The solution to this critical excitation problem provides a useful bound of the earthquake input energy for a class of ground motions satisfying intensity constraints (see Fig.14

including the plot for a record of Kobe Univ. NS during 1995 Hyogoken-Nanbu earthquake). The solution with acceleration constraints can bound properly the earthquake input energy in a shorter natural period range and that with velocity constraints can limit properly the input energy in an intermediate or longer natural period range. The applicability of this bound theory to various actual recorded ground motions can be found in Fig.15.

4. HIGH-RISE BUILDINGS AND BASE-ISOLATED BUILDINGS

The issue on long-period ground motions is a controversial one in Japan. This is because, although this type of ground motion has a small intensity in the scale of conventional intensity measure of ground motions, it could cause a large seismic demand to such structures as high-rise buildings, base-isolated buildings, oil tanks, etc. In these buildings under resonant long-period ground motions with long duration, not only large displacement demand but also large dissipation energy demand (fatigue problem) have to be resolved [26, 27]. The occurrence mechanisms of these long-period ground motions and its properties have been investigated extensively. However there remain a lot of uncertainties.

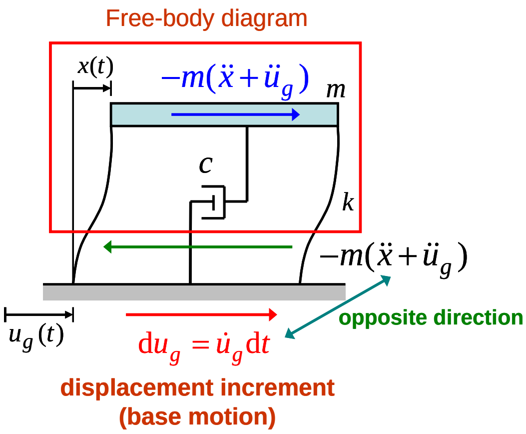


Fig.9 Free-body diagram for definition of earthquake input energy

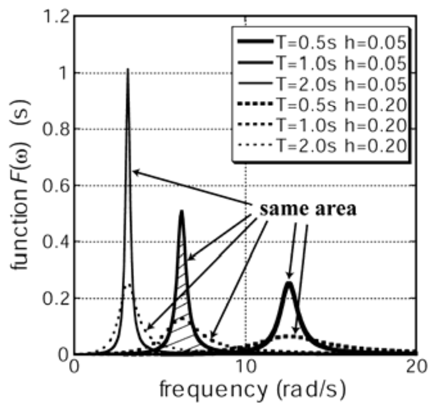


Fig.10 Energy transfer function with equi-area property for models of various natural periods and damping ratios

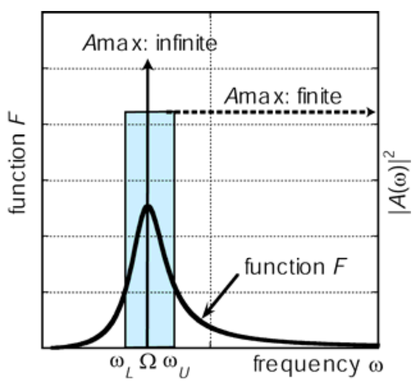


Fig.11 Schematic diagram of solution procedure for critical excitation problem with acceleration constraints

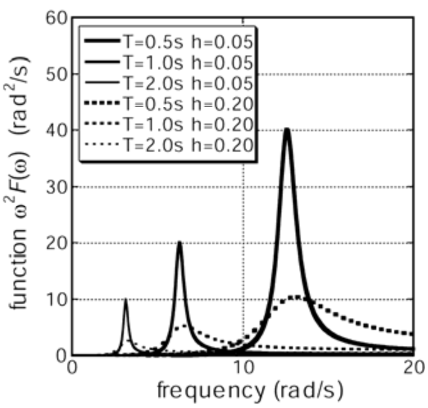


Fig.12 Energy transfer function for velocity input

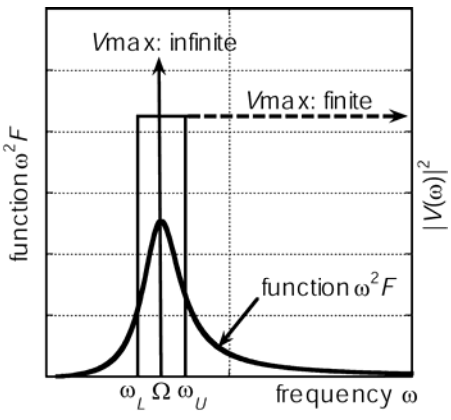


Fig.13 Schematic diagram of solution procedure for critical excitation problem with velocity constraints

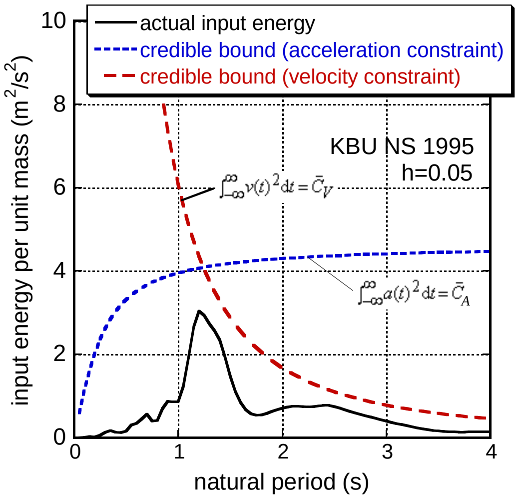


Fig.14 Credible bound of input energy for acceleration constraint and that for velocity constraint

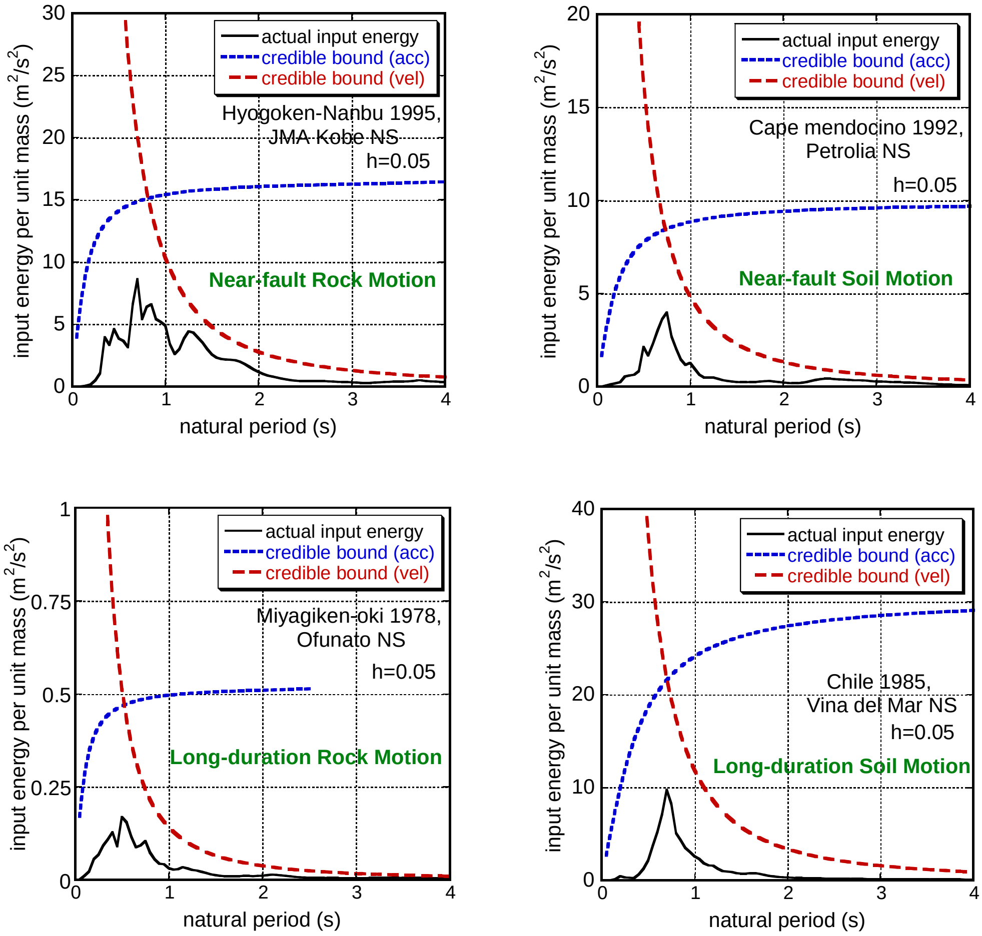
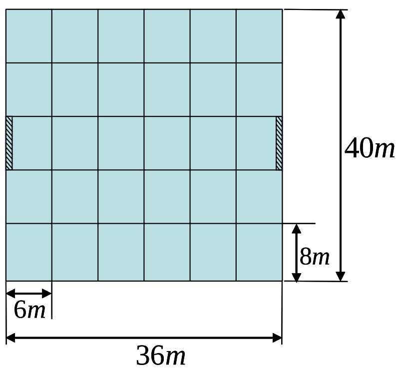
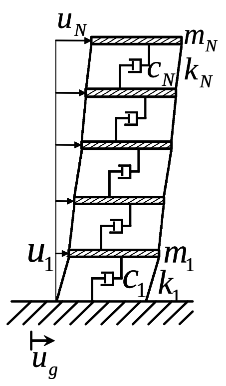


Fig.15 Credible bound of input energy for various types of ground motions (Near-fault rock motion, Near-fault soil motion, Long-duration rock motion, Long-duration soil motion)

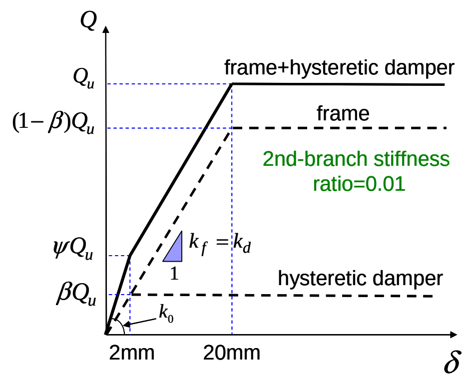
Consider two shear building models, as shown in Fig.16, including viscous or hysteretic dampers. It is known that both buildings have sinusoidal motions as an approximate critical excitation. Furtheremore, it is often the case that the long-period ground motions can be modeled approximately by sinusoidal motions. Fig.17 shows the 3-D sweeping response performance surfaces for the 5-storey building model with viscous dampers and that with hysteretic dampers. These building models have been reduced to two equivalent SDOF models. The maximum interstorey drift has been plotted with respect to the maximum acceleration amplitude and the excitation frequency. The left figure indicates the model with viscous dampers and the right figure shows the model with hysteretic dampers. The shape of these 3-D sweeping response performance surfaces implies the mechanical properties of these passive-controlled frames. Fig.18 shows the 2-D sweeping response performance curves for 5-storey models with viscous or hysteretic dampers and Fig.19 presents those for 20-storey models. The peak frequency shift with respect to increasing input intensities can be observed in these figures.



Plan view



Shear building model



Story shear-deformation relation
(hysteretic damper model)

Fig.16 Passively-controlled building model

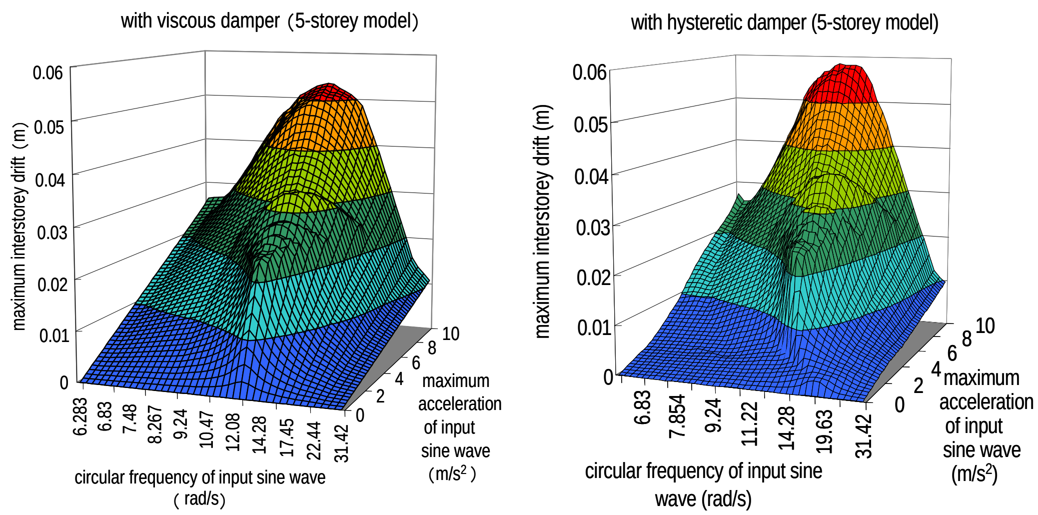


Fig.17 3-D sweeping response performance surface

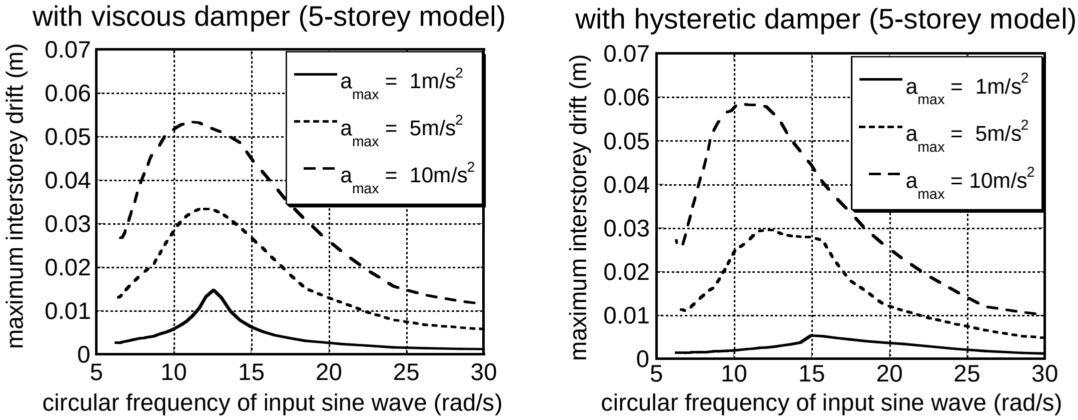


Fig.18 2-D sweeping response performance curves for 5-storey models with viscous or hysteretic dampers

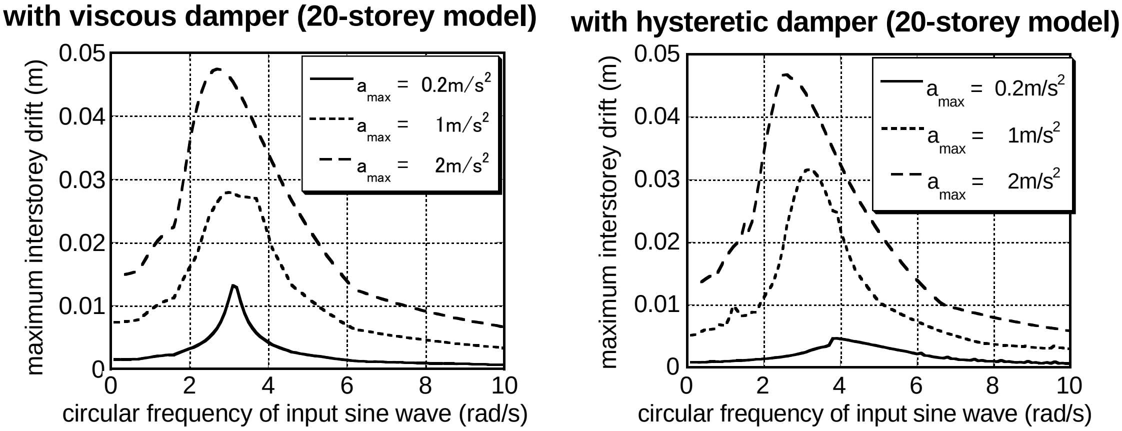
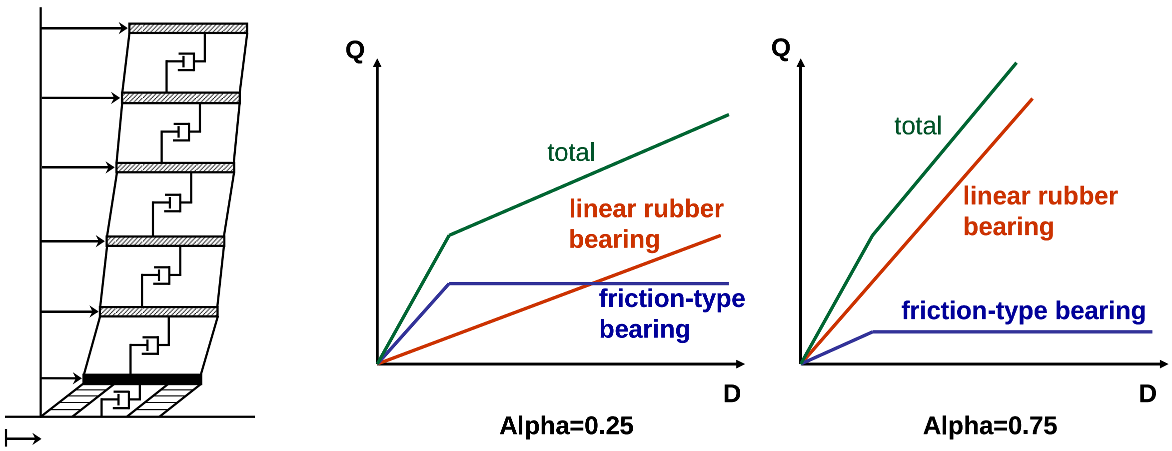


Fig.19 2-D sweeping response performance curves for 20-storey models with viscous or hysteretic dampers

The resonant behaviour of base-isolated high-rise buildings as shown in Fig.20(a) is investigated next [26] under long-period ground motions. The long-period ground motions are known to be induced by surface waves. While the acceleration amplitude of such long-period ground motion is small as stated before, the velocity amplitude is fairly large. It is expected that high-rise buildings and base-isolated buildings with long fundamental natural periods are greatly influenced by these long-period ground motions. Especially base-isolated high-rise buildings with friction-type bearings may have remarkable mechanical characteristics unfavourable for these long-period ground motions. The restoring-force characteristics of the isolation floor are shown in Fig.20(b). The parameter α indicates the ratio of the storey shear due to the linear rubber bearing to the total one at the displacement of initiation of slippage of friction dampers. The purpose here is to reveal that the long-period ground motions recorded in Japan have the intensity to make the base-isolated high-rise buildings in resonance with the long-period components and that careful treatment is inevitable in the structural design of these base-isolated high-rise buildings.

Figs.21 show the acceleration records of El Centro NS (Imperial Valley 1940), OSA NS (simulated Nankai Earthquake by Kamae et al.[28]), Tomakomai EW (Tokachi-oki 2003) and a simulated motion compatible with the response spectrum of Level 2 in the new Japanese earthquake-resistant design code (2000). Fig.22 presents the velocity response spectra of these four motions considered for damping ratio 0.05. Fig.23 shows the restoring-force characteristics in the base-isolation (BI) system under these four ground motions in the case of $\alpha = 0.25$. It can be observed that the ground motion of Tomakomai EW with long-period wave components has caused a dangerous result to base-isolated high-rise buildings (see Fig.24).



(a) (b)

Fig.20 (a) Base-isolated high-rise building, (b) Restoring-force characteristics of isolation floor

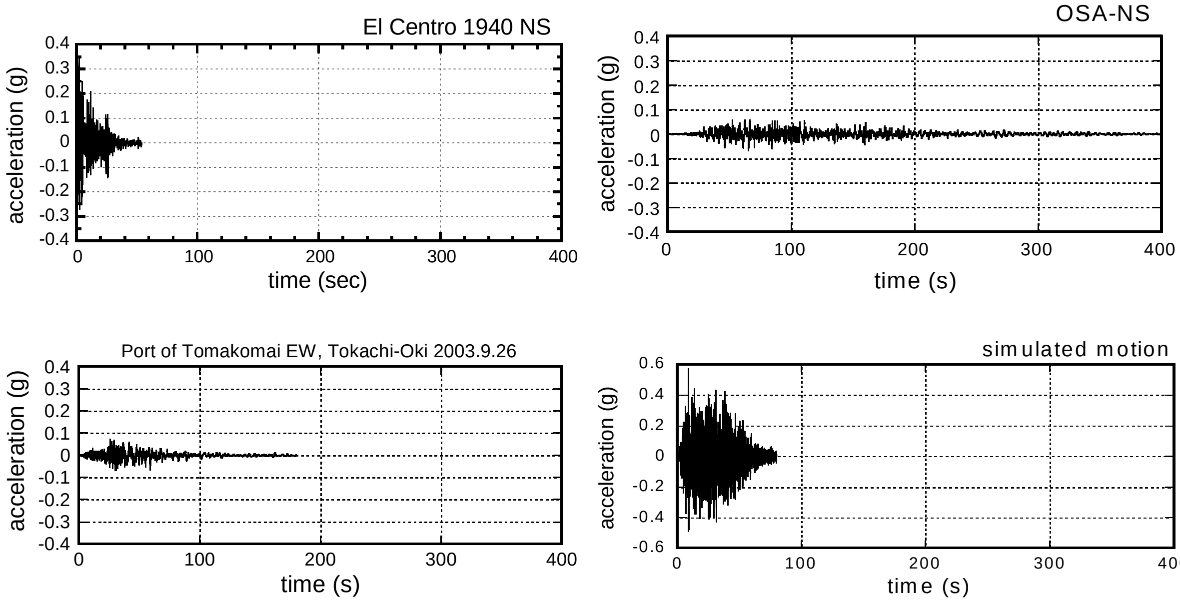


Fig.21 Acceleration records of El Centro NS (Imperial Valley 1940), OSA NS (simulated Nankai Earthquake by Kamae, Kawabe and Irikura 2004), Tomakomai EW (Tokachi-oki 2003) and a simulated motion compatible with the response spectrum of Level 2 in the new Japanese earthquake-resistant design code (2000)

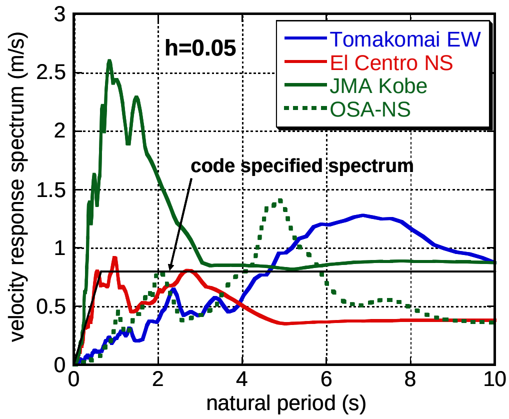


Fig.22 Velocity response spectra of four motions considered for damping ratio 0.05

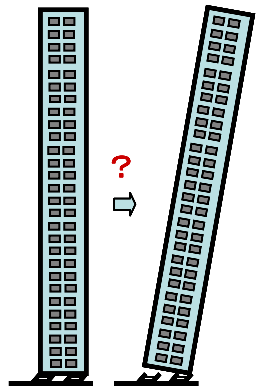


Fig.24 Limit state of rubber bearing in the tensile region

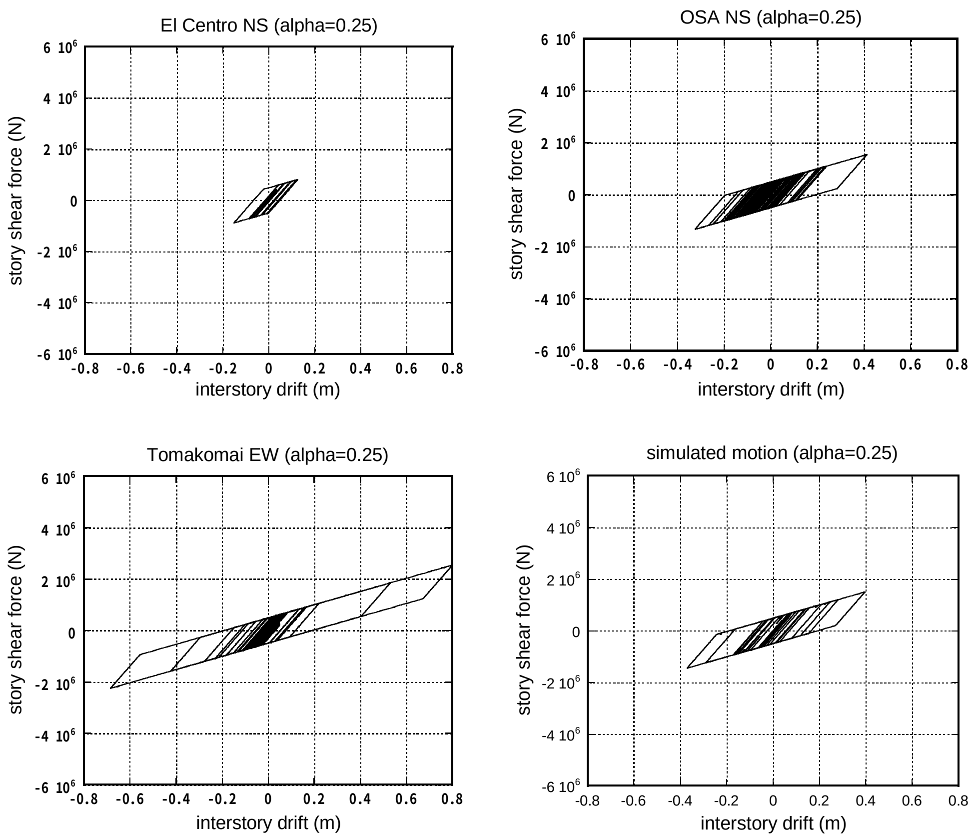


Fig.23 Restoring-force characteristics in the BI system under four ground motions in the case of $\alpha = 0.25$

The base-isolation systems with friction-type bearings are generally believed to be effective in providing damping in the BI system and avoiding resonance with ground motions by changing the equivalent natural frequency in accordance with experienced deformation. However resonance can occur in the case where the ground motion has a large intensity of velocity response spectra with broad band in the long natural period range. As the drift of the BI system becomes larger, the resonance is accelerated because the modified natural period is approaching the predominant period of the ground motion. This phenomenon is hard to occur in the high-rise buildings without BI system because such drastic change of the equivalent natural period is hard to occur even in the experience of minor plastic deformation in some parts.

As the damping ratio of the additional viscous-damper system in the BI system increases, most of the maximum interstorey drifts become smaller. However, the introduction of sufficient amount of damping in the BI system is very hard because of the limitation on space and cost. Furthermore, the maximum interstorey drifts and floor accelerations in upper stories under some ground motions can become larger in spite of the increase of the damping. Therefore special attention should be paid in the introduction of damping in the BI system.

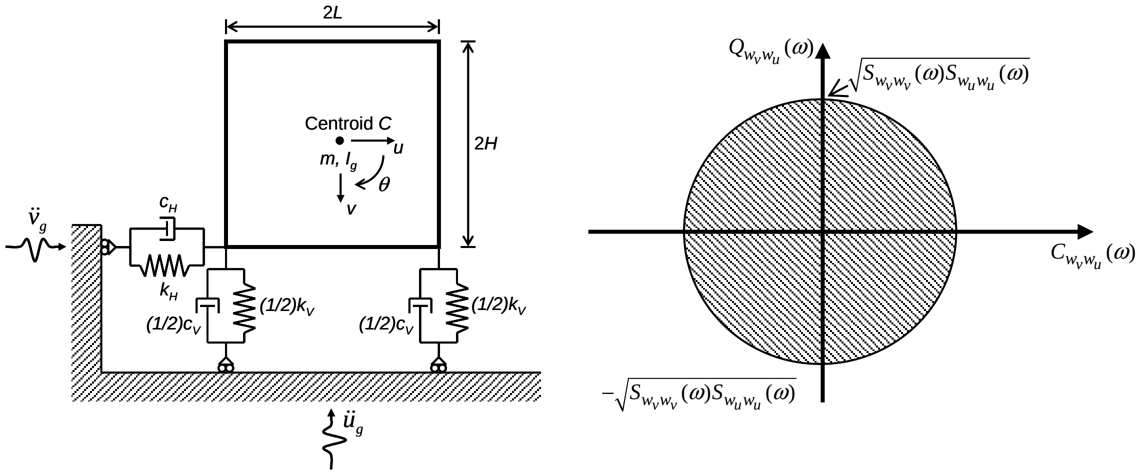
The empirical laws, known as the constant displacement or the constant energy, should be discussed from the view point of the relation of the natural period of structures with the predominant period of input ground motions. Over 40 years ago, most of the recorded ground motions have the predominant period mostly smaller than 1-2 sec. In such case, the structures with the fundamental natural period much larger than 1-2 sec tend to follow the constant displacement law. However, those structures do not necessarily follow the constant displacement law under the 'long-period earthquake ground motions'. The relation of the natural period of structures with the predominant period of input ground motions is a key to be discussed carefully.

5. NUCLEAR REACTOR FACILITIES

The ground surface motion during an earthquake consists of multiple components. It is often assumed that there exists a set of principal axes in the ground motions [29]. This set of principal axes varies in time. In the current seismic structural design practice, the effect of the multi-component inputs is often taken into account by use of the SRSS (square root of the sum of the squares) method in which the maximum responses under respective input motions are combined by the rule of SRSS. The SRSS method assumes the statistical independence among the maximum responses under respective input motions. However, the multi-component inputs have some statistical dependence (e.g. [30]). It seems that the discussion from the viewpoint of worst-case analysis is appropriate in such a case.

A problem of critical excitation is considered here for a rigid block subjected to horizontal and vertical simultaneous base inputs [31]. The rigid block is supported by a set of a horizontal spring and a dashpot and rests on another set of two vertical springs and dashpots as shown in Fig.25. These springs and dashpots are the representatives of the stiffness and damping of ground. The motion of the block consists of a horizontal-rotational (swaying-rocking) motion under a horizontal base input and a vertical motion under a vertical base input. Both the horizontal-rotational motion and the vertical motion clearly contribute to the uplift of the edge of the block. In the seismic structural design practice, the design spectra for the horizontal and vertical motions are often provided independently and it does not appear that their relationship is discussed in detail except a few (for example see [32]). This relationship will be discussed here from the viewpoint of critical excitation.

It seems natural and appropriate to assume that the horizontal and vertical base inputs are to be described by a non-stationary random process whose power spectra are prescribed. A critical excitation problem is considered such that the worst cross spectrum (allowable region is shown in Fig.25(b)) and the corresponding cross-correlation function of the horizontal and vertical inputs are searched for the maximum mean-squares response of the uplift. It is found that the real part (co-spectrum, e.g. see [33]) and the imaginary part (quad-spectrum) of the worst cross spectrum can be obtained by a devised algorithm, as shown in Fig.26, including the interchange of the double maximization procedure with respect to the time and cross-spectrum (*functional*: function of frequency) domains.



(a) (b)

Fig.25 (a)Elastically supported rigid block subjected to horizontal-vertical simultaneous inputs and (b) allowable domain defined from the property of the cross spectrum and the power spectra

The mean-squares uplift of the foundation edge may be described as

$$E[D(t)^2] = \int_{-\infty}^{\infty} F_{uu}(t;\omega) S_{w_u w_u}(\omega) d\omega + \int_{-\infty}^{\infty} F_{vv}(t;\omega) S_{w_v w_v}(\omega) d\omega + \int_{-\infty}^{\infty} F_{uv}(t;\omega) S_{w_u w_v}(\omega) d\omega \tag{5}$$

The cross spectrum is defined by

$$S_{w_u w_v}(\omega) = S_{w_v w_u}(\omega)^* = C_{w_v w_u}(\omega) + \mathrm{i} Q_{w_v w_u}(\omega) \tag{6}$$

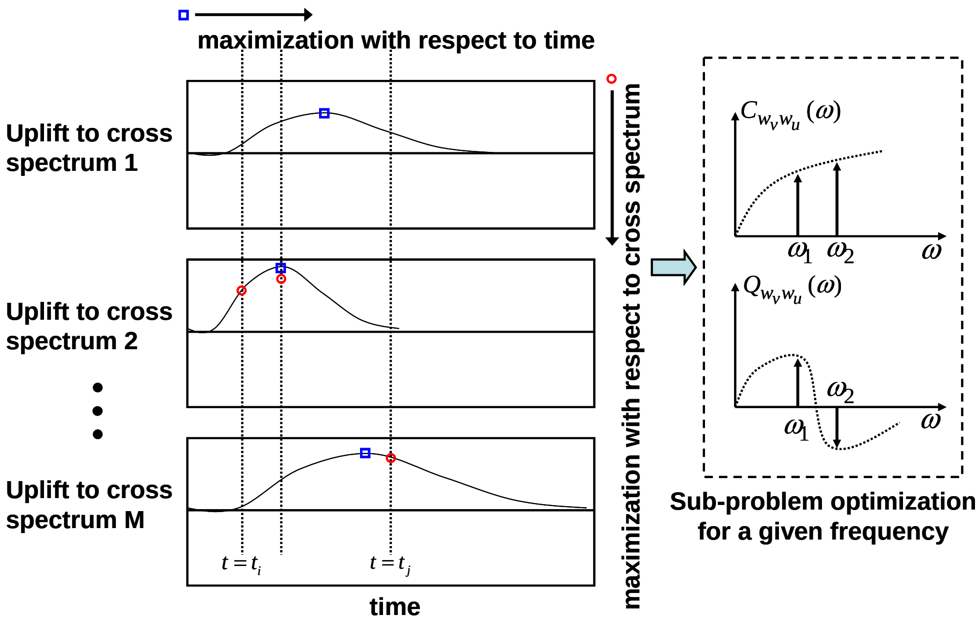


Fig.26 Schematic diagram of solution procedure for critical excitation problem with unknown cross-spectrum

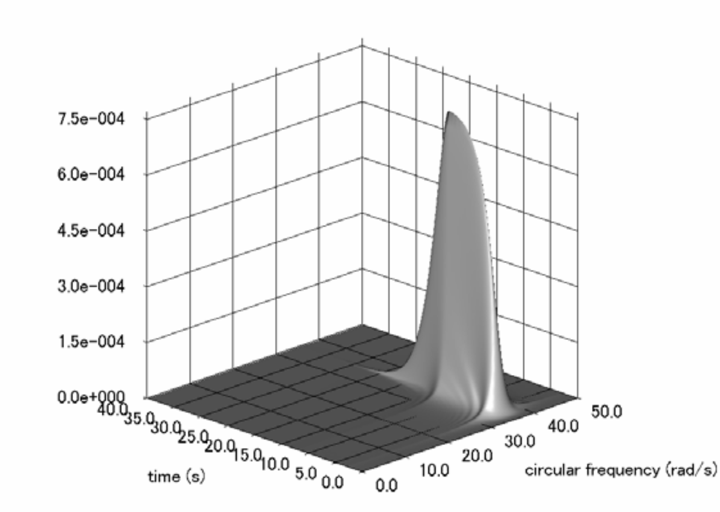
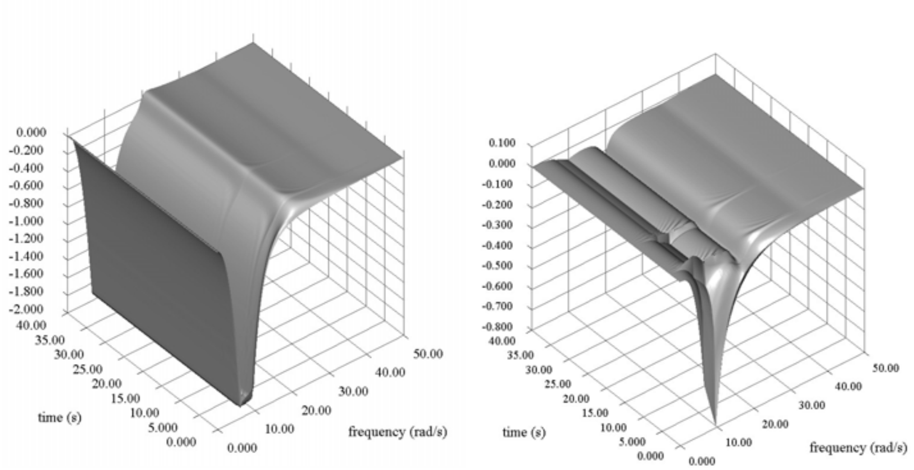
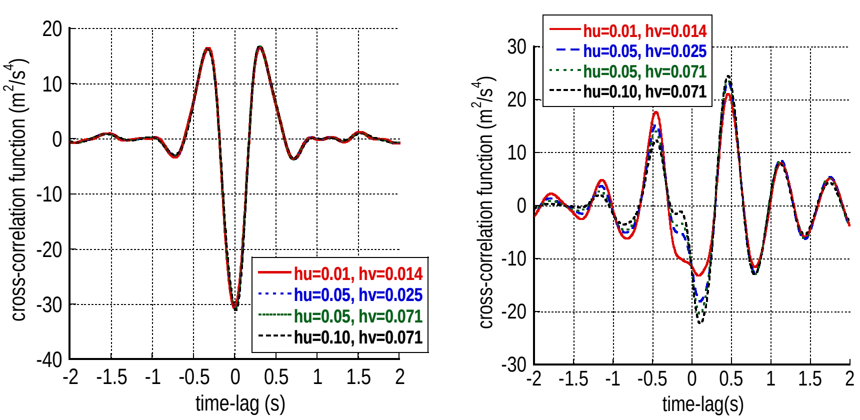


Fig.27 3-D view of the function, part of integrand for computation of mean-square-uplift under vertical input, with respect to time and frequency



(a) co-spectrum (b) quad-spectrum

Fig.28 3-D view of temporal characteristics of critical co-spectrum and quad-spectrum



(a) (b)

Fig.29 Critical cross-correlation function for $T_H = 0.25$(s) and $T_H = 0.50$(s)

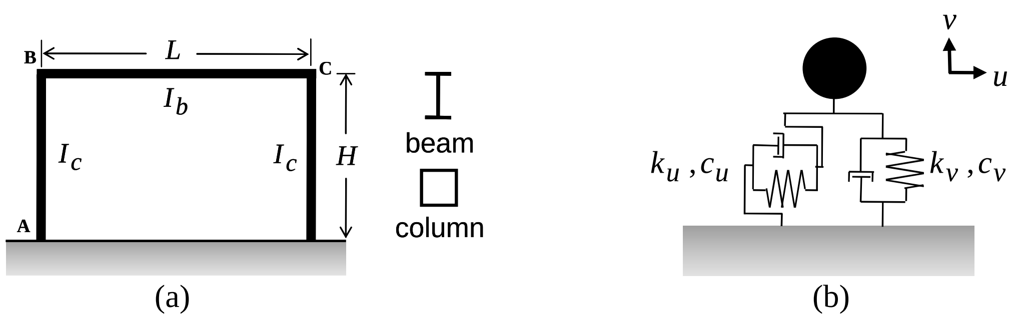
The mean-squares response of uplift has proven to be the sum of the term due to the horizontal input, that due to the vertical input and that due to their correlation as shown in Eq.(5). Since the power spectra of the horizontal and vertical ground motions are given and prescribed, the maximization in the critical excitation problem means the maximization of the correlation term of the horizontal and vertical ground motions.

The root-mean-squares value of uplift to the critical combination of the horizontal and vertical motions becomes about several through fifteen percent larger than that by the SRSS estimate of uplift due to the horizontal input and that due to the vertical input. The present method including the critical correlation of the horizontal and vertical inputs should be used in the design of important structures, e.g. nuclear reactor facilities, super high-rise buildings etc., in place of the well-known SRSS method.

In order to demonstrate the validity of the proposed critical excitation method, several numerical examples have been presented. Four damping cases have been treated which include proportional and non-proportional damping cases. The numerical examples indicate that the proposed algorithm can work very well and the critical cross spectrum and critical cross-correlation function can be understood from the view point of physical meaning. Fig.27 shows the 3-D view of the function, i.e. part of integrand for computation of mean-square-uplift under vertical input, with respect to time and frequency and Fig.28 presents the 3-D view of temporal characteristics of critical co-spectrum and quad-spectrum. When the damping ratio in the horizontal and vertical vibration modes is small, the critical cross-correlation function has its maximum at the zero time-lag as shown in Fig.29(a). On the other hand, when such a damping ratio is relatively large, the critical cross-correlation function has its maximum at a non-zero time-lag (see Fig.29(b)).

The present method has been applied to a problem of foundation uplift. However, the method can be applied to a broader class of critical excitation problems. For example, the present method has been applied to a moment-resisting frame of large span as shown in Fig.30(a) [34]. The horizontal and vertical stochastic ground accelerations are input to the frame and the mean-squares of the sum of the beam-end bending moments due to both inputs is adopted as the objective function for a critical excitation problem. The frame has been modelled into an SDOF system as shown in Fig.30(b).

Fig.31 shows the comparison of the maximum root mean squares of the sum of the beam-end bending moments among the SRSS estimate, the proportional input case (quad spectrum Q=0) and the critical input due to the present author [34]. It can be observed that, while the critical input is close to the proportional input case in the model (span length =17m) with an almost equal horizontal and vertical natural frequency, the critical input is far from the proportional input case in the other models.



(a) (b)

Fig.30 Moment-resisting frame of large span subjected to horizontal and vertical stochastic ground accelerations

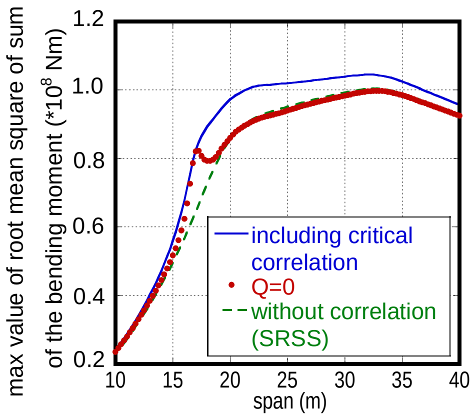


Fig.31 Comparison of the maximum RMS values of sum of beam-end bending moments among SRSS estimate, proportional input case (Q=0) and the critical input

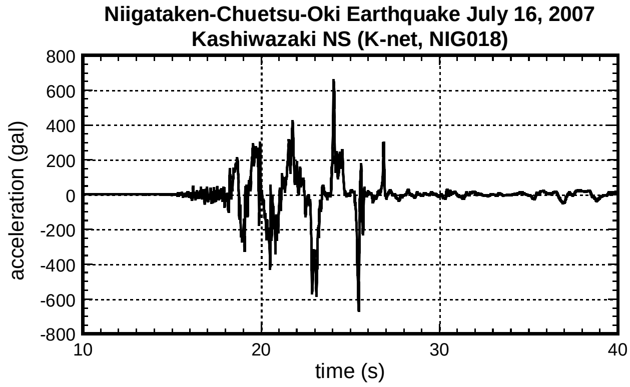


Fig.32 Resonant critical input in recent earthquake (near nuclear reactor facilities)

It may be said that the present theory opens the door for critical excitation problems for multi-component inputs. Although it is a rare case to include the effect of multi-component inputs in the current structural design practice, simultaneous consideration of bi-directional ground motion components is inevitable in the reliable design of structures. It has been demonstrated that the present model allowing arbitrary specification of the cross spectrum between respective-direction ground motion components includes the conventional Penzien-Watabe model assuming the existence of the principal axes without correlation and is suitable for seismic-resistant design of building structures. This is because the usual structural design practice requires the specification of design response spectra or power spectra in the building structural axes (usually one axis). The present theory satisfies this requirement and facilitates the introduction of the concept of critical excitation.

Recently (July 16, 2007), a severe ground motion attacked the city of Kashiwazaki, Niigata Prefecture in Japan and many old wood houses were destroyed. It has been reported that a peculiar ground motion as shown in Fig.32 has been observed and the ground motion had a predominant period of 2.5 (s). This period is thought to be resonant with the natural period of old wood houses with heavy roofs. This ground motion is very similar to one predicted in [35]. It should be noted that a large nuclear reactor facility is located in the city of Kashiwazaki and that facility had relatively minor damage. Further damage investigation is being conducted even now. While the IAEA reported that the damage is relatively minor, further extensive damage has been found at some areas. It is hoped that a critical excitation method is used in the design and upgrading of these nuclear reactor facilities.

6. CONCLUSIONS

(1) In critical excitation problems with time-dependent performance indices as objective functions, a frequency-domain approach is appropriate and a solution procedure consisting of the double maximization procedure can be developed. The double maximization procedure is with respect to the power spectrum density function and the time.

(2) In critical excitation problems for earthquake input energy, a frequency-domain approach is effective. The solution to this critical excitation problem provides a useful bound of the earthquake input energy for a class of ground motions satisfying intensity constraints. The solution with acceleration constraints can bound properly the earthquake input energy in a shorter natural period range and that with velocity constraints can limit properly the input energy in an intermediate or longer natural period range.

(3) Critical correlation among multi-component ground motions can be defined. This criticality reveals the amplification of the response over the conventional square root of the sum of the squares (SRSS) response.

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