

## 2 章 建築構造物のロバスト耐震設計のための極限入力解析

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### 研究発表

- [1] I.Takewaki, Robust building stiffness design for variable critical excitations, *Journal of Structural Engineering*, American Society of Civil Engineers, Vol.128, No.12, p1565-1574, 2002.
- [2] I.Takewaki, Bound of Earthquake Input Energy, *Journal of Structural Engineering*, ASCE, in press, 2004.
- [3] I.Takewaki, Frequency Domain Modal Analysis of Earthquake Input Energy to Highly Damped Passive Control Structures, *Earthquake Engineering & Structural Dynamics*, in press, 2004.
- [4] 竹脇 出, 多様な減衰分布を有する構造物に入力される地震エネルギーの限界値, 日本建築学会構造系論文集, 第 572 号, pp65-72, 2003.
- [5] I.Takewaki, Bound of Earthquake Input Energy to Soil-Structure Interaction Systems, *Proc. of The 11th International Conference on Soil Dynamics and Earthquake Engineering (ICSDEE) and the 3rd International Conference on Earthquake Geotechnical Engineering (ICEGE)*, Berkeley, January 7-9, 2004.

### アブストラクト

都市直下の内陸型地震において顕著な通り、これまでに観測されている地震動には、振幅・位相特性や卓越振動数特性等において大きな不確定性が存在しており、一定期間におけるその極めて小さな発生確率（長い再現期間）を考えると、近い将来にその不確定性が取り除かれ理論化されるというシナリオは描き難いと推測される。

このような状況下では、比較的不確定性のレベルが高い部分のみを未定パラメータとして含む地震動モデルを作成し、発生が予想される集合としての地震動群に対して構造物を安全に設計することが、現時点で最も期待できる信頼性の高い方法の一つといえる。本研究では、地震動のクリティカル性を特徴付ける指標として、構造物に入力されるエネルギーを新たに採用する。地震入力エネルギーの特性およびそれを用いた設計法に関しては、これまでに多くの成果が蓄積されている。本研究では、線形弾性応答に限定して、振動数領域における定式化により、あるクラスの地震動群に対してその上限値が誘導できることを示す。次に、構造物 地盤連成系をモデル化したスウェイ・ロッキングモデルに対して振動数領域定式化を適用することにより、基礎固定モデルと同様に地震入力エネルギーの上限値が誘導できることを示す。さらに、埋め込み基礎を有する場合の地震入力エネルギーの評価法についても論じる。

## 2. Worst-case Analysis for Robust Building Seismic Resistant Design

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### Abstract

A new general critical excitation method is developed for a damped linear elastic single-degree-of-freedom structure. In contrast to previous studies considering amplitude nonstationarity only, no special constraint of input motions is needed on nonstationarity. The input energy to the structure during an earthquake is introduced as a new measure of criticality. It is shown that the formulation of the earthquake input energy in the frequency domain is essential for solving the critical excitation problem and deriving a bound on the earthquake input energy for a class of ground motions. It is remarkable that no mathematical programming technique is required in the solution procedure. This enables structural engineers to use the method in their structural design practice without difficulty. The constancy of earthquake input energy for various natural periods and damping ratios is discussed based on an original sophisticated mathematical treatment. Through numerical examinations for four classes of recorded ground motions, the bounds under acceleration and velocity constraints (time integral of the squared base acceleration and time integral of the squared base velocity) are clarified to be meaningful in the short and intermediate/long natural period ranges, respectively. Another critical excitation method is also developed for a building structure supported by a sway-rocking system representing a foundation-ground system. It is shown that the frequency-domain approach is effective in developing a critical excitation method especially for a soil-structure interaction system.

### Introduction

Earthquake ground motions involve a lot of uncertain factors in the modeling of various aspects and it does not appear easy to predict forthcoming events precisely at a specific site both in time and frequency contents (see, for example, Abrahamson et al. 1998, Anderson and Bertero 1987, Geller et al. 1997; PEER Center et al. 2000; Stein 2003). Some of the uncertainties may result from the lack of information due to the low occurrence rate of large earthquakes and it does not seem that this problem can be resolved in the near future. Especially, the modeling of near-fault ground motions involves various uncertain factors in contrast to far-fault ground motions (Singh 1984; PEER Center et al. 2000; Krawinkler et al. 2001). It is therefore strongly desirable to develop a *robust* structural design method taking into account these uncertainties with limited information and enabling the design of safer structures for a broader class of design earthquakes.

To the best of the authors' knowledge, approaches based on the concept of "critical excitation" or "worst-case input" are promising (Drenick 1970; Shinozuka 1970; Takewaki 2002b). Evidently anticipated ground motions differ both in intensity and in character. It may be said that the critical excitation method is aimed at identifying the critical character. Just as the investigation of response limit states of structures plays an important role in specifying allowable response and performance limits of structures during disturbances, the clarification of critical excitations for a given structure appears to provide structural designers with useful information in determining excitation parameters in a reasonable and reliable way.

The critical excitation methods have a history of over 30 years. Its general review can be found in Takewaki (2002a). The previous studies have some limitations, e.g. those on treatment of nonstationarity of ground motions, those on numerical applicability.

One of the purposes of this research is to develop a new general critical excitation method for a damped linear elastic single-degree-of-freedom (SDOF) system. The input energy to the SDOF system during an earthquake is introduced as a new measure of criticality. It is shown that the formulation of the earthquake input energy in the frequency domain is essential for solving the critical excitation problem and deriving a bound on the earthquake input energy for a class of ground motions. The criticality is expressed in terms of degree of concentration of input motion components on the maximum portion of the characteristic function defining the earthquake input energy. It is remarkable that no mathematical programming technique is required in the solution procedure. The constancy of earthquake input energy (Housner 1956, 1959) for various natural periods and damping ratios is discussed from a new point of view based on an original sophisticated mathematical treatment. It is shown that the constancy of earthquake input energy is directly related to the uniformity of 'the Fourier amplitude spectrum' of ground motion acceleration, not the uniformity of the velocity response spectrum. The bounds under acceleration and velocity constraints (time integral of the squared base acceleration and time integral of the squared base velocity) are clarified through numerical examinations for recorded ground motions to be meaningful in the short and intermediate/long natural period ranges, respectively. Applicability of the proposed technique to a soil-structure interaction system is also discussed.

## Earthquake Input Energy to SDOF System

Much work has been accumulated on the topics of earthquake input energy (for example, Tanabashi 1935; Housner 1956, 1959; Berg and Thomaidis 1960; Goel and Berg 1968; Housner and Jennings 1975; Kato and Akiyama 1975; Takizawa 1977; Mahin and Lin 1983; Zahrah and Hall 1984; Akiyama 1985; Ohi et al. 1985; Uang and Bertero 1990; Leger and Dussault 1992; Kuwamura et al. 1994; Fajfar and Vidic 1994; Ogawa et al. 2000; Riddell and Garcia 2001; Ordaz et al. 2003). In contrast to most of the previous works, the earthquake input energy is formulated here in the frequency domain (Page 1952; Lyon 1975, Takizawa 1977; Ohi et al. 1985; Ordaz et al. 2003) to facilitate the introduction of critical excitation methods and the derivation of bound of earthquake input energy.

Consider a damped linear SDOF system of mass  $m$ , stiffness  $k$  and damping coefficient  $c$ . Let  $\Omega = \sqrt{k/m}$ ,  $h = c/(2\Omega m)$  and  $x$  denote the undamped natural circular frequency, the damping ratio and the displacement of the mass relative to the ground, respectively. Time derivative is denoted by over-dot. The input energy to an SDOF system by a uni-directional ground acceleration  $\ddot{u}_g(t) = a(t)$  from  $t=0$  to  $t=t_0$  (end of input) can be defined by the work of the ground on the structural system and is expressed by

$$E_I = \int_0^{t_0} m(\ddot{u}_g + \ddot{x})\dot{u}_g dt \quad (1)$$

The term  $-m(\ddot{u}_g + \ddot{x})$  indicates the inertial force and is equal to the sum of the restoring force  $kx$  and the damping force  $c\dot{x}$  in the system. Integration by parts of Eq.(1) provides

$$\begin{aligned} E_I &= \int_0^{t_0} m(\ddot{x} + \ddot{u}_g)\dot{u}_g dt = \int_0^{t_0} m\dot{x}\dot{u}_g dt + \left[ (1/2)m\dot{u}_g^2 \right]_0^{t_0} \\ &= \left[ m\dot{x}\dot{u}_g \right]_0^{t_0} - \int_0^{t_0} m\dot{x}\ddot{u}_g dt + \left[ (1/2)m\dot{u}_g^2 \right]_0^{t_0} \end{aligned} \quad (2)$$

If  $\dot{x}=0$  at  $t=0$  and  $\dot{u}_g=0$  at  $t=0$  and  $t=t_0$ , the input energy can be reduced to the following form.

$$E_I = -\int_0^{t_0} m\ddot{u}_g \dot{x} dt \quad (3)$$

It is known (Page 1952; Lyon 1975; Takizawa 1977; Ohi et al. 1985; Ordaz et al. 2003) that the input energy per unit mass can also be expressed in the frequency domain.

$$\begin{aligned} E_I / m &= -\int_{-\infty}^{\infty} \dot{x} a dt = -\int_{-\infty}^{\infty} \left[ (1/2\pi) \int_{-\infty}^{\infty} \dot{X} e^{i\omega t} d\omega \right] a dt \\ &= -(1/2\pi) \int_{-\infty}^{\infty} A(-\omega) \{ H_V(\omega; \Omega, h) A(\omega) \} d\omega \\ &= \int_0^{\infty} |A(\omega)|^2 \{ -\text{Re}[H_V(\omega; \Omega, h)] / \pi \} d\omega \\ &\equiv \int_0^{\infty} |A(\omega)|^2 F(\omega) d\omega \end{aligned} \quad (4)$$

where  $H_V(\omega; \Omega, h)$  is the transfer function defined by  $\dot{X}(\omega) = H_V(\omega; \Omega, h)A(\omega)$  and  $F(\omega) = -\text{Re}[H_V(\omega; \Omega, h)] / \pi$ .  $\dot{X}$  and  $A(\omega)$  are the Fourier transforms of  $\dot{x}$  and  $\ddot{u}_g(t) = a(t)$ , respectively. The symbol  $i$  denotes the imaginary unit.  $H_V(\omega; \Omega, h)$  can be expressed by

$$H_V(\omega; \Omega, h) = -i\omega / (\Omega^2 - \omega^2 + 2ih\Omega\omega) \quad (5)$$

Eq.(4) indicates that the earthquake input energy to damped linear elastic SDOF systems does not depend on the phase of input motions and this fact is well known (Page 1952, Lyon 1975, Takizawa 1977, Ohi et al. 1985, Kuwamura et al. 1994, Ordaz et al. 2003). It can also be understood from Eq.(4) that the function  $F(\omega)$  plays an important role in the evaluation of earthquake input energy and may have some influence on the investigation of constancy of earthquake input energy for structures with various model parameters. The property of the function  $F(\omega)$  defined in Eq.(4) will therefore be clarified in the following section.

## Property of Energy Transfer Function $F(\omega)$ and Constancy of Earthquake Input Energy

The functions  $F(\omega)$  for various natural periods  $T=0.5, 1.0, 2.0$ s and damping ratios  $h=0.05, 0.20$  are plotted in Fig.1. It is interesting to note that the area of  $F(\omega)$  can be proved to be constant regardless of  $\Omega$  and  $h$ . This fact for any damping ratio has already been pointed out by Ordaz et al. (2003). However, its proof has never been presented. The proof is shown here.

The function  $F(\omega)$ , called the energy transfer function, can be expressed by

$$F(\omega) = \frac{2h\Omega\omega^2}{\pi\{(\Omega^2 - \omega^2)^2 + (2h\Omega\omega)^2\}} \quad (6)$$

Four singular points of the function  $F(\omega)$  in terms of complex variables can be obtained as

$$z_1 = (hi + \sqrt{1-h^2})\Omega, \quad z_2 = (hi - \sqrt{1-h^2})\Omega, \quad z_3 = (-hi + \sqrt{1-h^2})\Omega, \quad z_4 = (-hi - \sqrt{1-h^2})\Omega.$$

Consider an integration path in the complex plane as shown in Fig.2. The singular points inside the integration path are  $z_1$  and  $z_2$ . The residues for the singular points  $z_1$  and  $z_2$  can be computed as

$$\text{Res}[z = z_1] = \frac{2hz_1^2\Omega}{\pi(z_1 - z_2)(z_1 - z_3)(z_1 - z_4)} = \frac{z_1}{4\pi\sqrt{1-h^2}\Omega i} \quad (7a)$$

$$\text{Res}[z = z_2] = \frac{2hz_2^2\Omega}{\pi(z_2 - z_1)(z_2 - z_3)(z_2 - z_4)} = \frac{-z_2}{4\pi\sqrt{1-h^2}\Omega i} \quad (7b)$$

The integration path consists of one on the real axis and the other on the semi-circle. The integral for the path on the semi-circle will vanish as the radius becomes infinite. On the other hand, the integral on the real axis with infinite lower and upper limits corresponding to infinite radius will be reduced to the residue theorem. The residue theorem provides

$$\int_{-\infty}^{\infty} F(\omega)d\omega = 2\pi i \times (\text{Res}[z = z_1] + \text{Res}[z = z_2]) \quad (8)$$

Substitution of Eqs.(7a, b) into Eq.(8) and the property of  $F(\omega)$  as an even function lead to the following relation.

$$2\int_0^{\infty} F(\omega)d\omega = 1 \quad (9)$$

Eq.(9) indicates that the area of  $F(\omega)$  is constant regardless of  $\Omega$  and  $h$ .

It can be stated from Eqs.(4) and (9) that, if the Fourier amplitude spectrum of input ground acceleration is uniform with respect to frequency, the earthquake input energy to a damped linear SDOF system per unit mass is exactly constant regardless of natural frequency and damping ratio. Let  $S_V(h=0)$  denote the velocity response spectrum for null damping ratio. If  $A(\omega)$  is exactly constant with respect to frequency and an assumption  $A(\Omega) \cong S_V(h=0)$  (Hudson 1962) holds, Eqs.(4) and (9) lead to

$$E_I \cong \frac{1}{2}m\{S_V(h=0)\}^2 \quad (10)$$

Eq.(10) is similar to the maximum total energy proposed by Housner (1956, 1959). It is noted that Housner (1959) discussed the maximum total energy defined by  $E_H = \max_t \{-\int_0^t m\ddot{u}_g \dot{x} dt\}$  instead of  $E_I$  defined by Eq.(3) and introduced some assumptions, e.g. slow variation of the total energy.  $S_V(h \neq 0) \leq S_V(h=0)$  and a more exact relation  $A(\Omega) \leq S_V(h=0)$  can also be shown for most cases. If  $S_V(h \neq 0) \cong A(\Omega) = \text{constant}$  holds for a specific damping ratio  $h_a$ , Eq.(10) may be replaced by  $E_I \cong (1/2)m\{S_V(h_a)\}^2$  in better approximation. While Housner discussed the constancy of earthquake input energy (maximum total energy) only with respect to natural period by paying special attention to the uniformity of velocity response spectrum with respect to natural period (Housner 1956, 1959), another view point based on sophisticated mathematical treatment has been introduced in this paper. It should be noted that the constancy of earthquake input energy defined by Eq.(3) is *directly* related to the uniformity of 'the Fourier amplitude spectrum' of ground motion acceleration, not the uniformity of the velocity response spectrum. This problem will be investigated numerically for some recorded ground motions afterwards.

## Critical Excitation Problem for Earthquake Input Energy with Acceleration Constraint

It is shown in this section that a critical excitation method for earthquake input energy can provide upper bounds on earthquake input energy. Westermo (1985) has discussed a similar

problem for the maximum input energy to an SDOF system subjected to external forces. His solution is restrictive because it is of the form including the velocity response quantity containing the solution itself implicitly. A more general solution procedure will be presented here.

The capacity of ground motions is often defined in terms of the time integral of squared ground acceleration (Arias 1970; Housner and Jennings 1975; Riddell and Garcia 2001). This quantity is well known as the Arias intensity measure except a difference in the coefficient. The constraint on this quantity can be expressed by

$$\int_{-\infty}^{\infty} a(t)^2 dt = (1/\pi) \int_0^{\infty} |A(\omega)|^2 d\omega = \bar{C}_A \quad (11)$$

where  $\bar{C}_A$  is the specified value of the time integral of squared ground acceleration. It is also clear that the maximum value of the Fourier amplitude spectrum of input ground acceleration is finite. The infinite spectrum may correspond to a perfect harmonic function or that multiplied by an exponential function (Drenick 1970) which is unrealistic as an actual ground motion. The constraint on this property may be described by

$$|A(\omega)| \leq \bar{A} \quad (\bar{A}: \text{specified value}) \quad (12)$$

The critical excitation problem may be stated as follows:

### **Critical excitation problem for acceleration**

Find  $|A(\omega)|$  that maximizes the earthquake input energy per unit mass, Eq.(4), subject to the constraints (11) and (12) on ground acceleration.

It is clear from the present author's work (Takewaki 2001a, b, 2002b) on power spectral density functions that, if  $\bar{A}$  is infinite,  $|A(\omega)|^2$  turns out to be the Dirac's delta function which has a non-zero value at the point maximizing  $F(\omega)$ . On the other hand, if  $\bar{A}$  is finite,  $|A(\omega)|^2$  yields a rectangular function attaining  $\bar{A}^2$ . The band-width of the frequency can be obtained as  $\Delta\omega = \pi\bar{C}_A / \bar{A}^2$ . The position of the rectangular function, i.e. the lower and upper limits, can be computed by maximizing  $\bar{A}^2 \int_{\omega_L}^{\omega_U} F(\omega) d\omega$ . It is noted that  $\omega_U - \omega_L = \Delta\omega$ . It can be shown that a good and simple approximation can be made by  $(\omega_U + \omega_L)/2 = \Omega$ . The essential feature of the solution procedure presented in this section is shown in Fig.3. It is interesting to note that Westermo's periodic solution (Westermo 1985) may correspond to the case of infinite  $\bar{A}$ .

The absolute maximum (absolute bound) may be computed for the infinite value  $\bar{A}$ . This absolute maximum can be evaluated as  $\bar{C}_A / (2h\Omega)$  by employing a reasonable assumption that  $F(\omega)$  attains its maximum at  $\omega = \Omega$  and substituting Eq.(11) into Eq.(4).

### **Critical Excitation Problem for Earthquake Input Energy with Velocity Constraint**

It is often argued that the maximum ground acceleration controls the behavior of structures with short natural periods and the maximum ground velocity does the behavior of structures with intermediate or rather long natural periods (see, for example, Tanabashi 1956). On the basis of this argument, consider the following constraint on ground motion velocity  $\dot{u}_g(t) = v(t)$ .

$$\int_{-\infty}^{\infty} v(t)^2 dt = (1/\pi) \int_0^{\infty} |V(\omega)|^2 d\omega = \bar{C}_V \quad (\bar{C}_V: \text{specified value}) \quad (13)$$

where  $V(\omega)$  is the Fourier transform of the ground motion velocity. From the relation  $A(\omega) = i\omega V(\omega)$ , Eq.(4) can be reduced to

$$E_I / m = \int_0^{\infty} |V(\omega)|^2 \omega^2 F(\omega) d\omega \quad (14)$$

It is clear that the maximum value of  $|V(\omega)|$  is finite in a realistic situation. The constraint on the upper limit on  $V(\omega)$  may be described by

$$|V(\omega)| \leq \bar{V} \quad (\bar{V} : \text{upper limit of } V(\omega)) \quad (15)$$

The critical excitation problem for velocity constraints may be stated as follows:

### **Critical excitation problem for velocity**

Find  $|V(\omega)|$  that maximizes the earthquake input energy per unit mass, Eq.(14), subject to the constraints (13) and (15) on ground velocity.

It is clear that almost the same solution procedure as for acceleration constraints can be used by replacing  $A(\omega)$  and  $F(\omega)$  by  $V(\omega)$  and  $\omega^2 F(\omega)$ , respectively. The functions  $\omega^2 F(\omega)$  for various natural periods  $T=0.5, 1.0, 2.0$ s and damping ratios  $h=0.05, 0.20$  are plotted in Fig.4. It can be observed that  $\omega^2 F(\omega)$  becomes larger in peak and wider with increase in natural frequency. In case of a finite value  $\bar{V}$ , the frequency band-width of the critical rectangular function  $|V(\omega)|^2$  can be derived from  $\Delta\omega = \pi\bar{C}_V / \bar{V}^2$ . The upper and lower limits of the rectangular function can be specified by maximizing  $\bar{V}^2 \int_{\omega_L}^{\omega_U} \omega^2 F(\omega) d\omega$  where  $\omega_U - \omega_L = \Delta\omega$ . A good and simple approximation can be obtained by employing  $(\omega_U + \omega_L)/2 = \Omega$ . The essential feature of the solution procedure presented in this section is shown in Fig.5.

The absolute maximum (absolute bound) may be computed for the infinite value  $\bar{V}$ . This absolute maximum can be evaluated as  $\Omega\bar{C}_V / (2h)$  by employing an assumption that  $\omega^2 F(\omega)$  attains its maximum at  $\omega = \Omega$  and substituting Eq.(13) into Eq.(14).

## **Actual Earthquake Input Energy and its Bound for Recorded Ground Motions**

In order to investigate the distance from actual input energy of upper bound of earthquake input energy presented in the foregoing sections, numerical calculation has been conducted for some recorded ground motions. The ground motions were chosen from the PEER motions (Abrahamson et al. 1998). Four types of ground motions, i.e. (1) one at rock site in near-fault earthquake (near-fault rock motion), (2) one at soil site in near-fault earthquake (near-fault soil motion), (3) one of long-duration at rock site (long-duration rock motion) and (4) one of long-duration at soil site (long-duration soil motion). The profile of the selected motions is shown in Table 1. The Fourier amplitude spectra of these motions (acceleration) are plotted in Figs.6(a)-(d).  $A_{\max} = \max|A(\omega)|$  and  $V_{\max} = \max|V(\omega)|$  have been used as  $\bar{A}$  and  $\bar{V}$ , respectively. Due to this treatment of  $\bar{A}$  and  $\bar{V}$ , the bounds, shown in the previous sections, for acceleration and velocity constraints are called 'credible bounds' in the following. The selection of  $\bar{A}$  and  $\bar{V}$  may be arguable. It is clear at least that, if  $\bar{A}$  is chosen between  $A_{\max} = \max|A(\omega)|$  and infinity, the corresponding bound of earthquake input energy lies between the credible bound and the absolute maximum (absolute bound)  $\bar{C}_A / (2h\Omega)$ . A similar fact can be stated. If  $\bar{V}$  is chosen between  $V_{\max} = \max|V(\omega)|$  and infinity, the corresponding bound of earthquake input energy lies between the credible bound and the absolute maximum (absolute bound)  $\Omega\bar{C}_V / (2h)$ . The quantities  $A_{\max}, \bar{C}_A, \Delta\omega$  corresponding to the critical excitation problem for acceleration constraints are shown in Table 2 and those  $V_{\max}, \bar{C}_V, \Delta\omega$  corresponding to the critical excitation problem for velocity constraints are shown in Table 3.

Fig.7(a) presents the actual earthquake input energy for various natural periods and its corresponding credible bounds for near-fault rock motions. The damping ratio is fixed to 0.05. It can be observed that, since the Fourier amplitude spectrum of ground acceleration is not uniform even in the frequency range of interest in almost all the ground motions, the constancy of earthquake input energy is not seen in the present case. As for the bound of input energy, it is interesting to note that the monotonic increase of credible bound for acceleration constraints in the shorter natural period range results mainly from the characteristic of the function  $F(\omega)$  as a monotonically increasing function with respect to natural period (Fig.1 is arranged with respect to natural frequency). This fact explains mathematically actual phenomena for most ground motions. It can also be observed that the actual input energy in the shorter natural period range is bounded properly by the bound for acceleration constraints and that in the intermediate and longer natural period range is bounded properly by the bound for velocity constraints. These properties correspond well to the well-known fact (Tanabashi 1956) that the maximum ground acceleration influences the behavior of structures with shorter natural periods and the maximum ground velocity controls the behavior of structures with intermediate or longer natural periods. From another point of view, it may be said from Fig.7(a) that, while the behavior of structures with shorter natural periods is governed by an hypothesis of ‘constant energy’, that of structures with intermediate or longer natural periods is governed by a hypothesis of ‘constant maximum displacement’. In the previous studies on earthquake input energy, this property of ‘constant maximum displacement’ in the longer natural period range has never been considered explicitly.

Fig.7(b) shows the actual earthquake input energy for various natural periods and its corresponding credible bounds for near-fault soil motions. As in near-fault rock motions, the actual input energy is bounded properly by the two kinds of bound. It is also clear that most of the bounds in the intermediate natural period range are nearly constant. This fact can be explained by Eqs.(4), (9), (11) and the critical shape of  $|A(\omega)|^2$  as rectangular one. It is noted that the frequency limits  $\omega_L$  and  $\omega_U$  are varied so as to coincide with the peak of  $F(\omega)$  for varied natural period.

Fig.7(c) shows those for long-duration rock motions and Fig.7(d) illustrates those for long-duration soil motions. It can also be observed that the actual input energy is bounded properly by the two kinds of bound pointed out earlier. It may be concluded that two kinds of bound proposed in this paper provide a physically meaningful unified limit on earthquake input energy for various types of recorded ground motions.

## Robust Design Problem

Consider an  $n$ -story shear building model supported by swaying and rocking springs and dashpots. The set of story stiffnesses is denoted by  $\mathbf{k} = \{k_i\}$ . The design problem treated here may be stated as:

### **Design problem for critical state**

Find  $\mathbf{k} = \{k_i\}$  and  $|A(\omega)|$

$$\text{such that } \min_{\mathbf{k}} \left( \max_{|A(\omega)|} E_I^S \right) \quad (16)$$

subject to

$$\int_{-\infty}^{\infty} a(t)^2 dt = (1/\pi) \int_0^{\infty} |A(\omega)|^2 d\omega = \bar{C} \quad (17)$$

$$|A(\omega)| \leq \bar{A} \quad (18)$$

$$\sum_{i=1}^n k_i = \bar{K} \quad (19)$$

$$k_i > 0 \quad (i = 1, \dots, n) \quad (20)$$



## Solution Procedure for Robust Design Problem

Let  $\tilde{\Omega} = \Delta\omega$  denote the frequency bandwidth in the positive frequency range of the critical rectangular function  $|A(\omega)|$ . It is assumed here that the upper and lower frequency limits of the rectangular Fourier amplitude spectrum of the input acceleration can be expressed by

$$\omega_U = \omega_1 + (1/2)\tilde{\Omega}, \quad \omega_L = \omega_1 - (1/2)\tilde{\Omega} \quad (21)$$

where  $\bar{A}^2\tilde{\Omega} = \pi\bar{C}$ . The input energy to the structure corresponding to the critical input may be described by

$$\hat{E}_I^S = \int_0^\infty F_S(\omega; \mathbf{k}) |A(\omega)|^2 d\omega \equiv \bar{A}^2 \int_{\omega_L}^{\omega_U} F_S(\omega; \mathbf{k}) d\omega = \bar{A}^2 \{ \Phi(\omega_U; \mathbf{k}) - \Phi(\omega_L; \mathbf{k}) \} \quad (22)$$

where  $\Phi(\bar{\omega}; \mathbf{k}) = \int_0^{\bar{\omega}} F_S(\omega; \mathbf{k}) d\omega$ . This insightful approximate manipulation enables an analytical treatment of the present complicated strongly nonlinear problem.

Let us define the following Lagrange function in terms of a Lagrange multiplier  $\lambda$ .

$$L = \hat{E}_I^S + \lambda \left( \sum_{i=1}^n k_i - \bar{K} \right) = \bar{A}^2 \{ \Phi(\omega_U; \mathbf{k}) - \Phi(\omega_L; \mathbf{k}) \} + \lambda \left( \sum_{i=1}^n k_i - \bar{K} \right) \quad (23)$$

The stationarity condition of the Lagrange function with respect to story stiffnesses may be described by

$$\begin{aligned} \partial L / \partial k_i &= \hat{E}_{I,i}^S + \lambda \\ &= \bar{A}^2 \left[ (\partial \omega_1 / \partial k_i) \{ F_S(\omega_U; \mathbf{k}) - F_S(\omega_L; \mathbf{k}) \} \right. \\ &\quad \left. + \int_{\omega_L}^{\omega_U} (\partial F_S(\omega; \mathbf{k}) / \partial k_i) d\omega \right] + \lambda = 0 \end{aligned} \quad (24)$$

where  $\hat{E}_{I,i}^S \equiv \partial \hat{E}_I^S / \partial k_i$  and Eq.(24) represents the optimality condition.

This robust design problem can be solved by almost the same procedure developed in the reference (Takewaki 2002b) which is based on the incremental inverse problem due to the present author.

## Earthquake Input Energy to SDOF System with Embedded Foundation

Consider a linear elastic SDOF super-structure of story stiffness  $k$  and story damping coefficient  $c$ , as shown in Fig.8, with a cylindrical rigid foundation embedded in the uniform half-space ground. Let  $r_0$  and  $e$  denote the radius and the depth of the foundation, respectively. Let  $m$  and  $I_R$  denote the mass of the super-structure and the mass moment of inertia of the super-structure and let  $m_0$  and  $I_{R0}$  denote the mass of the embedded foundation and the mass moment of inertia of the embedded foundation "around its top center node". The height of the super-structure mass from the ground surface is denoted by  $H$ .

$U_0^*$  and  $\Theta_0^*$  are the effective input motions in the frequency domain for horizontal and rotational components, respectively, at the top center of the foundation. The corresponding effective input motions in the time domain may be expressed by

$$\dot{u}_0^*(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \dot{U}_0^*(\omega) e^{i\omega t} d\omega = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_{HT}(\omega) \ddot{U}_g(\omega) e^{i\omega t} d\omega \quad (25a)$$

$$\ddot{\theta}_0^*(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \ddot{\Theta}_0^*(\omega) e^{i\omega t} d\omega = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_{RT}(\omega) \ddot{U}_g(\omega) e^{i\omega t} d\omega \quad (25b)$$

$S_{HT}(\omega)$  and  $S_{RT}(\omega)$  are the ratios of the effective input motions,  $U_0^*$  and  $\Theta_0^*$ , in the frequency domain for horizontal and rotational components, respectively, at the top center of the foundation to the Fourier transform  $U_g(\omega)$  of the free-field horizontal ground-surface displacement. Let us assume that only a vertically incident shear wave (SH wave) is considered.

$S_{HT}(\omega)$  and  $S_{RT}(\omega)$  are expressed in terms of the ratios  $S_{HB}(\omega)$  and  $S_{RB}(\omega)$ , given in Meek and Wolf (1994) and Wolf (1994), of the effective input motions in the frequency domain for horizontal and rotational ( $\times r_0$ ) components at the bottom center of the foundation to  $U_g(\omega)$ .

$$S_{HT}(\omega) = S_{HB}(\omega) + (e/r_0)S_{RB}(\omega) \quad (26a)$$

$$S_{RT}(\omega) = (1/r_0)S_{RB}(\omega) \quad (26b)$$

The frequency-domain formulation of the earthquake input energy to the soil-structure interaction system can be found in the reference (Takewaki and Fujimoto 2003).

Examples of time histories of earthquake input energies (no embedment of foundation) and their final values  $E_I^A$ ,  $E_I^S$  for El Centro NS (Imperial Valley 1940) and Kobe University NS (Hyogoken-Nanbu 1995) are shown in Fig.9 for natural period of the structure  $T=0.5$ (s) and ground shear wave velocity  $V_s=50, 100$ (m/s). A similar concept has been proposed by Yang and Akiyama (2000).

Fig.10 shows the earthquake input energies to the structure-foundation-soil system  $E_I^A$  and the structure only  $E_I^S$  with various degrees of foundation embedment  $e/r_0 = 0.0, 0.5, 1.0, 2.0$  for the ground equivalent shear wave velocity=100(m/s) to El Centro NS of Imperial Valley 1940. It can be observed from Fig.10 that, while the input energy to the structure alone is smaller than that to the structure-foundation-soil system in all the natural period range up to 2.0(s) for  $e/r_0 = 0.0, 2.0$ , that relation does not exist for  $e/r_0 = 0.5, 1.0$ . More detailed examination for a broader range of parameters will be necessary to clarify the effect of degree of foundation embedment on the input energies to a structure and to the corresponding structure-foundation-soil system.

It should be remarked that only once computation of the Fourier amplitude spectra of ground motion accelerations is necessary and the input energy can be evaluated by combining those, through numerical integration in the frequency domain, with the energy transfer function. The structural designers can understand easily approximate input energies from the relation of the Fourier amplitude spectra of ground motion accelerations with the energy transfer functions both of which are expressed in the frequency domain.

The solid lines in Fig.11 show the plots of earthquake input energies by the ground motion of El Centro NS to the overall system (structure plus surrounding soil) without embedment of foundation and the structure alone for  $V_s=50, 100, 200$ (m/s) with respect to natural period of the fixed-base structure. The damping ratio of the super-structure is 0.05. It can be observed from Fig.11 that the input energy to stiff structures with short natural periods is governed primarily by the energy dissipated by the ground (surrounding soil) and the input energy to flexible structures with intermediate natural periods (around 1(s)) is governed mainly by the energy dissipated by the damping of super-structures. This phenomenon corresponds well to the well-known fact that the soil-structure interaction effect is notable in the stiff structures on flexible ground. The dotted lines in Fig.11 show the credible and absolute bounds of earthquake input energies by the ground motion of El Centro NS to the overall system and the structure alone. As the shear wave velocity of the ground becomes larger, the input energy is governed mainly by the energy dissipated by the damping of super-structures. It should also be pointed out that the ground motion of El Centro NS does not have a notable predominant period and the distance between the actual input energy and the credible bound is almost constant with respect to natural period of the super-structure.

## Conclusions

The conclusions may be stated as follows.

- (1) The function  $F(\omega)$  characterizing the earthquake input energy in the frequency domain to a damped linear elastic SDOF system has been proved to have an equi-area property regardless of natural period and damping ratio. This property guarantees that, if the Fourier amplitude spectrum of ground motion acceleration is uniform with respect to frequency, the constancy of earthquake input energy defined by Eq.(3) holds strictly. Otherwise, its constancy is not guaranteed. It should be remarked that the constancy of earthquake input energy defined by Eq.(3) is *directly* related to the uniformity of ‘the Fourier amplitude spectrum’ of ground motion acceleration, not the uniformity of the velocity response spectrum.
- (2) A new critical excitation method has been formulated which has the earthquake input energy as a new measure of criticality and has acceleration and/or velocity constraints (time integral of squared base acceleration and time integral of squared base velocity). No mathematical programming technique is needed in this method and structural engineers can find the solution without difficulty.
- (3) The solution to the aforementioned critical excitation problem provides a useful bound of the earthquake input energy for a class of ground motions satisfying intensity constraints. The solution with acceleration constraints can bound properly the earthquake input energy in a shorter natural period range and that with velocity constraints can bound properly the earthquake input energy in an intermediate or longer natural period range.
- (4) A new critical excitation method has been developed for soil-structure interaction systems. Definition of two input energies, one to the overall system (structure plus surrounding soil) and the other to the structure alone is very useful in understanding the mechanism of energy input and the effect of soil-structure interaction under various conditions of soil properties and natural period of structures.
- (5) Even soil-structure interaction systems including embedded foundations can be treated in a simple way and effects of the foundation embedment on the earthquake input energy to the super-structure can be clarified systematically by the proposed frequency domain formulation. It can be stated from a limited analysis that the input energy to the sway-rocking model without embedment is almost the same as that to the fixed-base model. As the degree of embedment becomes larger, the input energy is decreased regardless of the natural period range. The ratio of the input energy to the structure alone to that to the structure-foundation-soil system is affected in a complicated manner by the degree of embedment.

The evaluation of the earthquake input energy in the time domain is suitable for the evaluation of the time history of the input energy, especially for non-linear systems. Dual use of the frequency-domain and time-domain techniques may be preferable in the advanced seismic analysis for robust design.

## Appendix I. References

- Abrahamson, N., Ashford, S., Elgamal, A., Kramer, S., Seible, F., and Somerville, P. (1998). *Proceedings of 1st PEER Workshop on Characterization of Special Source Effects*.
- Akiyama, H. (1985). *Earthquake Resistant Limit-State Design for Buildings*. University of Tokyo Press, Tokyo, Japan.
- Anderson, J.C., and Bertero, V.V. (1987). "Uncertainties in establishing design earthquakes." *Journal of Structural Engineering*, ASCE, 113(8), 1709-1724.
- Arias, A. (1970). "A measure of earthquake intensity." In *Seismic Design for Nuclear Power Plants* (Ed. R.J.Hansen), The MIT Press, Cambridge, MA, 438-469.
- Berg, G.V., and Thomaides T.T. (1960). "Energy consumption by structures in strong-motion earthquakes." *Proceedings of 2nd World Conf. on Earthquake Engineering*, Tokyo and Kyoto, 681-696.

- Drenick, R.F. (1970). "Model-free design of aseismic structures." *Journal of Engineering Mechanics Division*, ASCE, 96(EM4), 483-493.
- Fajfar, P., and Vidic, T. (1994). "Consistent inelastic design spectra: hysteretic and input energy." *Earthquake Engineering and Structural Dynamics*, 23(5), 523-537.
- Geller, R.J., Jackson, D.D., Kagan, Y.Y., and Mulargia, F. (1997). "Earthquakes cannot be predicted." *Science*, 275, 1616.
- Goel, S.C., and Berg, G.V. (1968). "Inelastic earthquake response of tall steel frames." *Journal of Structural Division*, ASCE, 94, 1907-1934.
- Housner, G.W. (1956). "Limit design of structures to resist earthquakes." *Proceedings of the First World Conference on Earthquake Engineering*, Berkeley, CA, 5:1-11.
- Housner, G.W. (1959). "Behavior of structures during earthquakes." *Journal of the Engineering Mechanics Division*, ASCE, 85(4), 109-129.
- Housner, G.W., and Jennings, P.C. (1975) "The capacity of extreme earthquake motions to damage structures." *Structural and geotechnical mechanics: A volume honoring N.M.Newmark* edited by W.J. Hall, 102-116, Prentice-Hall Englewood Cliff, NJ.
- Hudson, D.E. (1962). "Some problems in the application of spectrum techniques to strong-motion earthquake analysis." *Bulletin of Seismological Society of America*, 52, 417-430.
- Kato, B., and Akiyama, H. (1975). "Energy input and damages in structures subjected to severe earthquakes." *Journal of Structural and Construction Eng.*, Archi. Inst. of Japan, 235, 9-18 (in Japanese).
- Krawinkler, H., Medina, R., and Alavi, B. (2001). "Seismic drift and ductility demands and their dependence on ground motions." *Proc. of US-Japan seminar on advanced stability and seismicity concepts for performance-based design of steel and composite structures*, Kyoto, Japan, July 23-26.
- Kuwamura, H., Kirino, Y., and Akiyama, H. (1994). "Prediction of earthquake energy input from smoothed Fourier amplitude spectrum." *Earthquake Engineering and Structural Dynamics*, 23, 1125-1137.
- Leger, P., and Dussault, S. (1992). "Seismic-energy dissipation in MDOF structures." *Journal of Structural Engineering*, ASCE, 118(5) 1251-1269.
- Lyon, R.H. (1975). *Statistical energy analysis of dynamical systems*, The MIT Press, Cambridge, MA.
- Mahin, S.A., and Lin, J. (1983). "Construction of inelastic response spectrum for single-degree-of-freedom system." *Report No. UCB/EERC-83/17*, Earthquake Engineering Research Center, Univ. of California, Berkeley, CA.
- Meek, J.W., Wolf, J.P. (1994) "Cone models for embedded foundation." *J Geotechnical Engng*, ASCE, 120(1), 60-80.
- Ohi, K., Takanashi, K., and Tanaka, H. (1985). "A simple method to estimate the statistical parameters of energy input to structures during earthquakes." *Journal of Structural and Construction Eng.*, Archi. Inst. of Japan, 347, 47-55 (in Japanese).
- Ogawa, K., Inoue, K., and Nakashima, M. (2000). "A study on earthquake input energy causing damages in structures." *Journal of Structural and Construction Eng.*, Archi. Inst. of Japan, 530, 177-184 (in Japanese).
- Ordaz, M., Huerta, B., and Reinoso, E. (2003). "Exact computation of input-energy spectra from Fourier amplitude spectra" *Earthquake Engineering and Structural Dynamics*, 32, 597-605.
- Page, C.H. (1952). "Instantaneous power spectra." *Journal of Applied Physics*, 23(1), 103-106.
- PEER center, ATC, Japan Ministry of education, science, sports, and culture, US-NSF (2000). *Effects of near-field earthquake shaking*, *Proc. of US-Japan workshop*, San Francisco, March 20-21, 2000.
- Riddell, R., and Garcia, J.E. (2001). "Hysteretic energy spectrum and damage control." *Earthquake Engineering and Structural Dynamics*, 30, 1791-1816.

- Shinozuka, M. (1970). "Maximum structural response to seismic excitations." *Journal of Engineering Mechanics Division*, ASCE, 96(EM5), 729-738.
- Singh, J.P. (1984). "Characteristics of near-field ground motion and their importance in building design." *ATC-10-1 Critical aspects of earthquake ground motion and building damage potential*, ATC, 23-42.
- Stein, R.S. (2003). "Earthquake conversations." *Scientific American*, 288(1), 72-79.
- Takewaki, I. (2001a). "A new method for nonstationary random critical excitation." *Earthquake Engineering and Structural Dynamics*, 30(4), 519-535.
- Takewaki, I. (2001b). "Probabilistic critical excitation for MDOF elastic-plastic structures on compliant ground." *Earthquake Engineering and Structural Dynamics*, 30(9), 1345-1360.
- Takewaki, I. (2002a). "Critical excitation method for robust design: A review." *Journal of Structural Engineering*, ASCE, 128(5), 665-672.
- Takewaki, I. (2002b). "Robust building stiffness design for variable critical excitations." *Journal of Structural Engineering*, ASCE, 128(12) 1565-1574.
- Takewaki, I. and Fujimoto, H. (2003). "Earthquake input energy to soil-structure interaction systems: a frequency-domain approach." *Int. J. of Advances in Structural Engineering*, submitted.
- Takizawa, H. (1977). "Energy-response spectrum of earthquake ground motions." *Proc. of the 14th Natural disaster science symposium*, 359-362, Hokkaido (in Japanese).
- Tanabashi, R. (1935). "Personal view on destructiveness of earthquake ground motions and building seismic resistance." *Journal of Architecture and Building Science*, Archi. Inst. of Japan, 48(599) (in Japanese).
- Tanabashi, R. (1956). "Studies on the nonlinear vibrations of structures subjected to destructive earthquakes." *Proceedings of the First World Conference on Earthquake Engineering*, Berkeley, CA, 6:1-16.
- Uang, C.M., and Bertero, V.V. (1990). "Evaluation of seismic energy in structures." *Earthquake Engineering and Structural Dynamics*, 19, 77-90.
- Westermo, B.D. (1985). "The critical excitation and response of simple dynamic systems." *Journal of Sound and Vibration*, 100(2), 233-242.
- Wolf, J.P. (1994) *Foundation vibration analysis using simple physical models*. Prentice-Hall, Englewood Cliffs, NJ.
- Yang, Z and Akiyama, H. (2000). "Evaluation of soil-structure interaction in terms of energy input." *Journal of Structural and Construction Eng.*, Archi. Inst. of Japan, 536, 39-45 (in Japanese).
- Zahrah, T.F., and Hall, W.J. (1984). "Earthquake energy absorption in SDOF structures." *Journal of Structural Engineering*, ASCE, 110(8) 1757-1772.

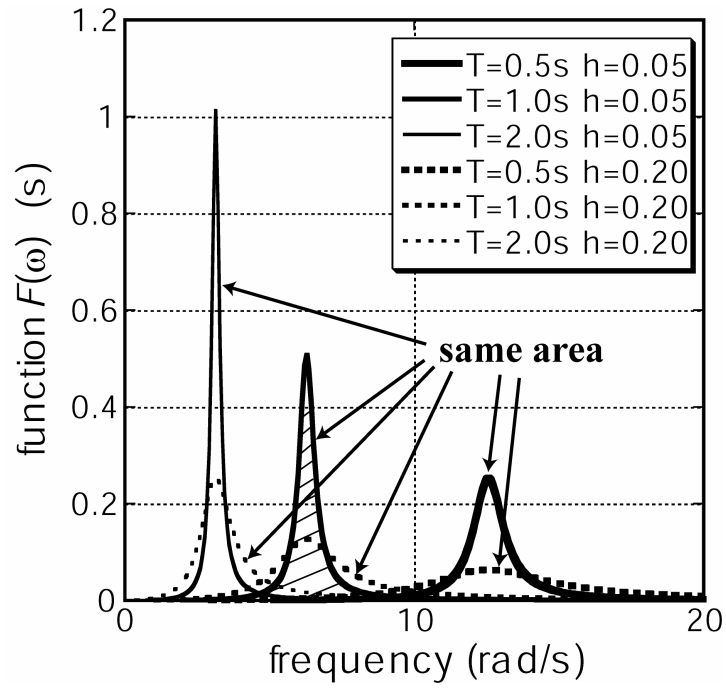


Fig.1 Energy transfer function  $F(\omega)$  for natural periods  $T=0.5, 1.0, 2.0$ s and damping ratios  $h=0.05, 0.20$

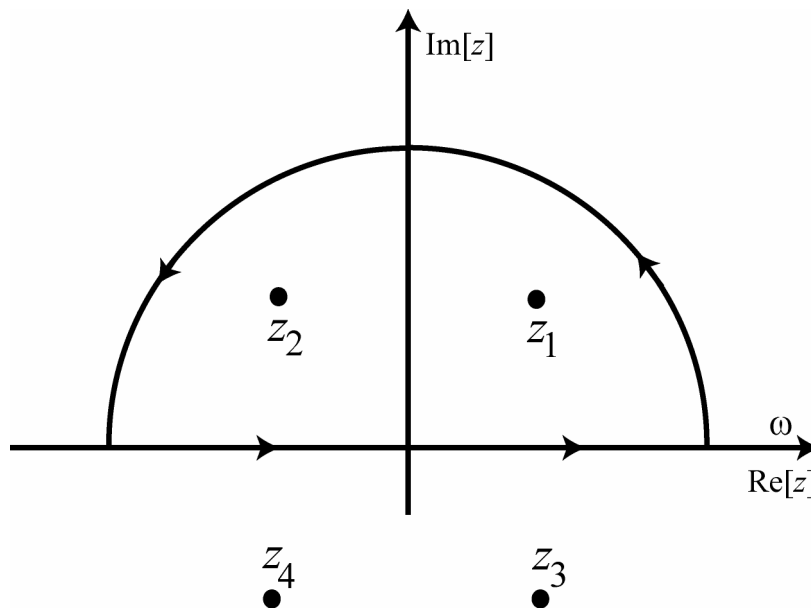


Fig.2 Integration path in complex plane and singular points  $z_1, z_2$  of function  $F(\omega)$  inside the path and those  $z_3, z_4$  outside the path

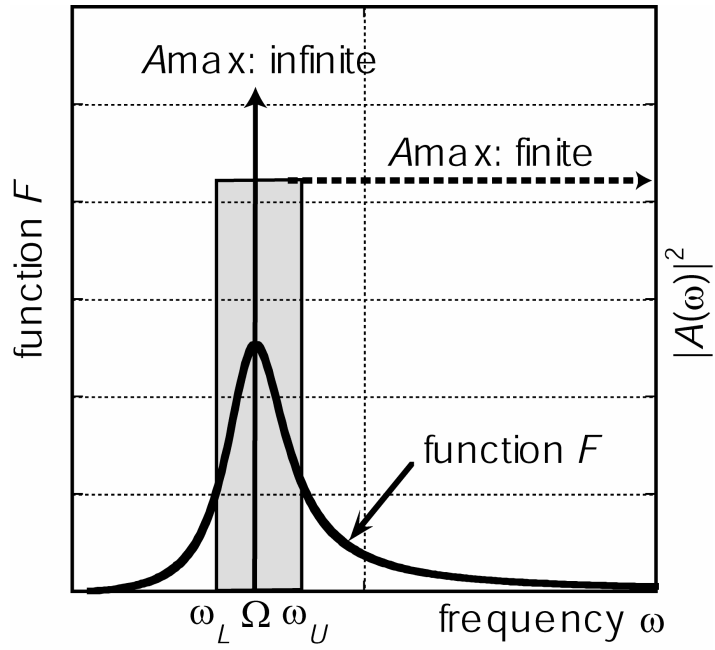


Fig.3 Schematic diagram of solution procedure for critical excitation problem with acceleration constraints

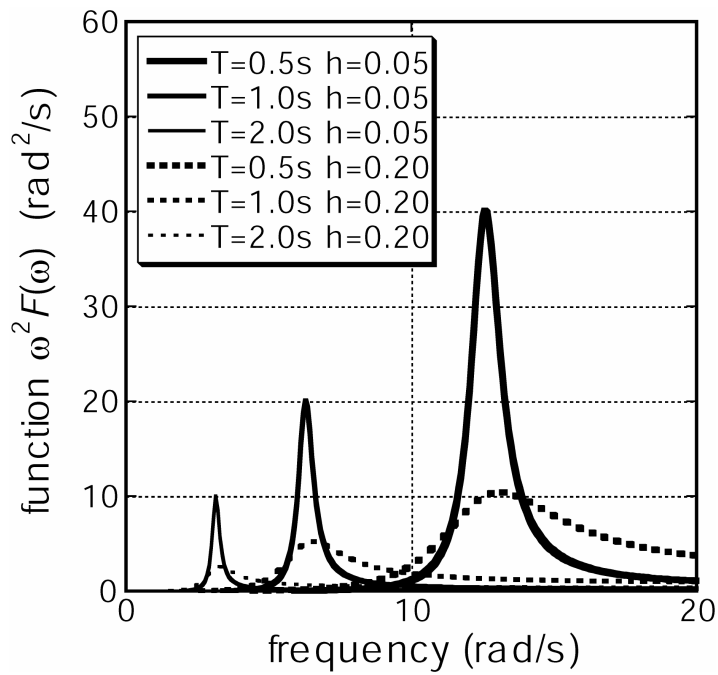


Fig.4 Function  $\omega^2 F(\omega)$  for natural periods  $T=0.5, 1.0, 2.0s$  and damping ratios  $h=0.05, 0.20$

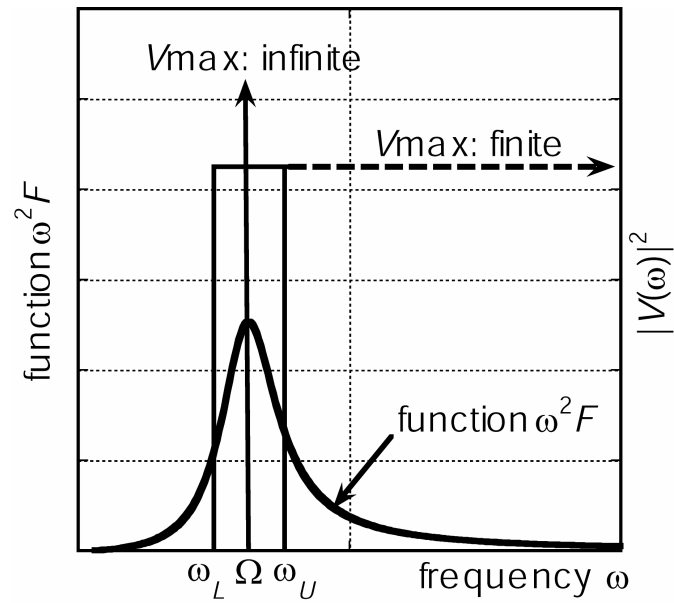


Fig.5 Schematic diagram of solution procedure for critical excitation problem with velocity constraints

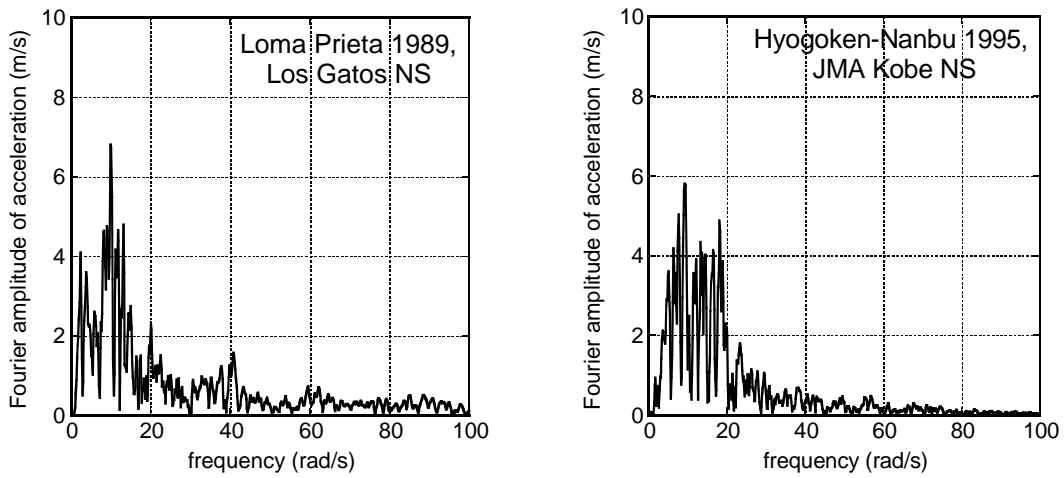


Fig.6(a) Near-fault rock motion

Fig.6 Fourier amplitude spectrum of ground motion acceleration:

- (a) Near-fault rock motion,
- (b) Near-fault soil motion,
- (c) Long-duration rock motion,
- (d) Long-duration soil motion



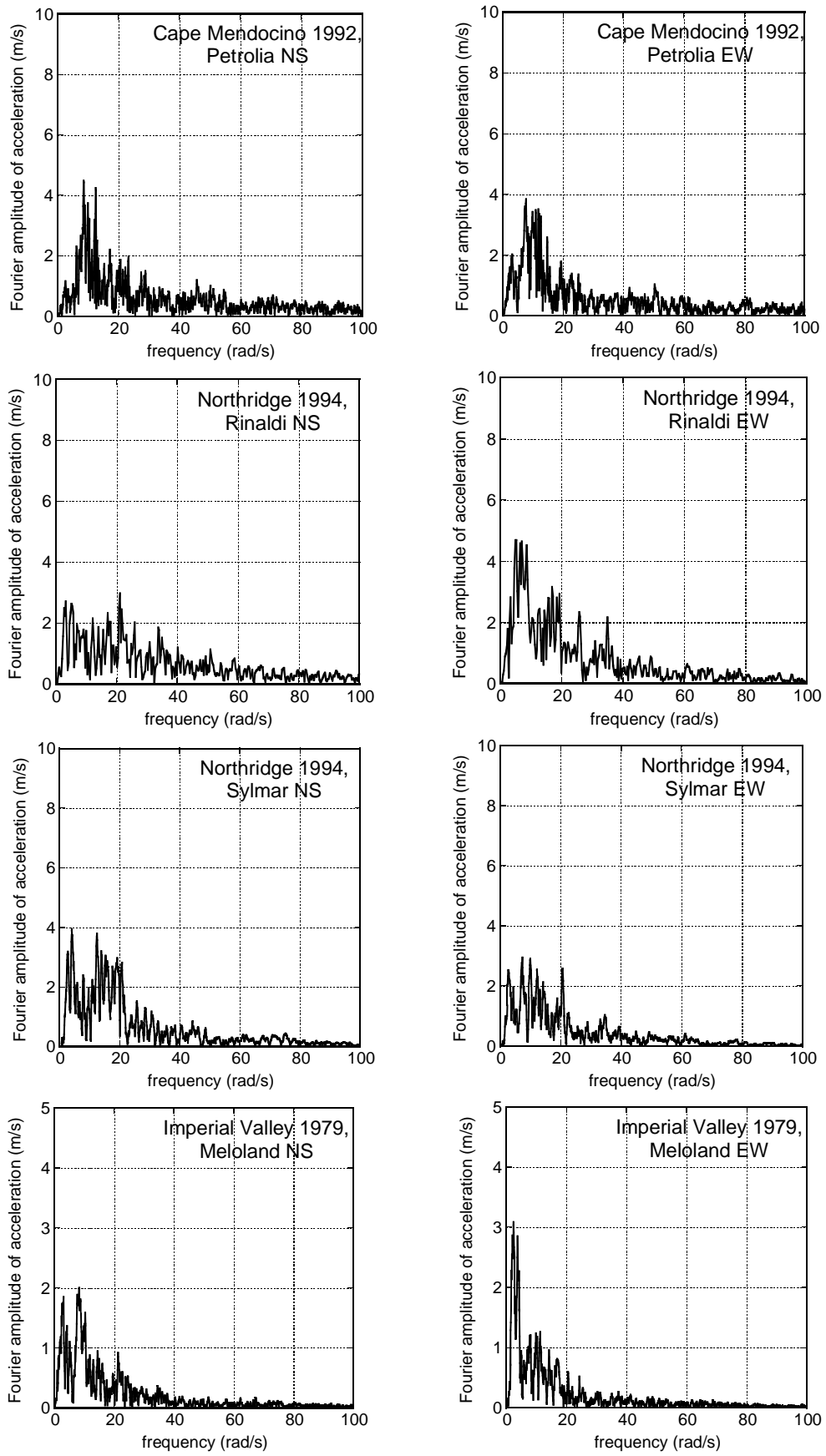


Fig.6(b) Near-fault soil motion

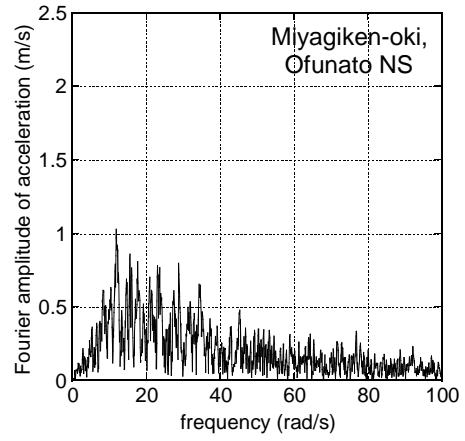
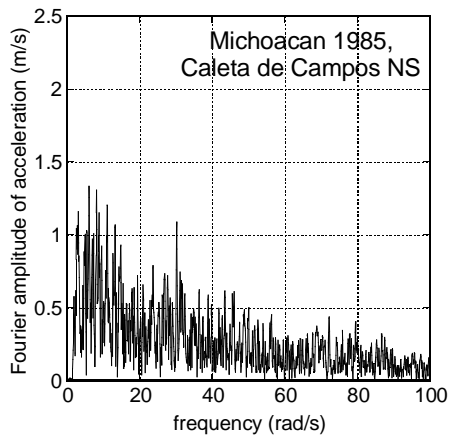


Fig.6(c) Long-duration rock motion

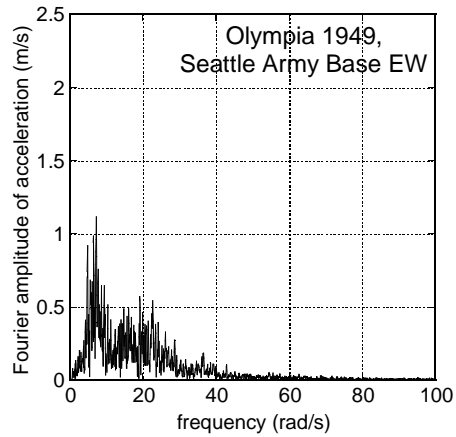
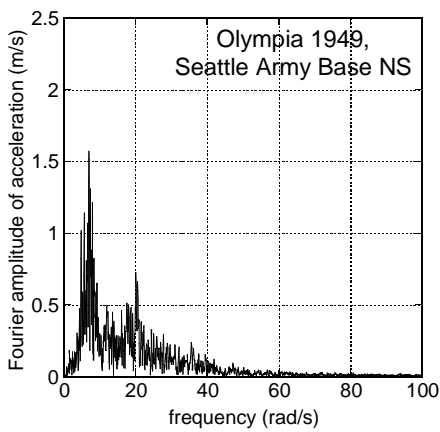
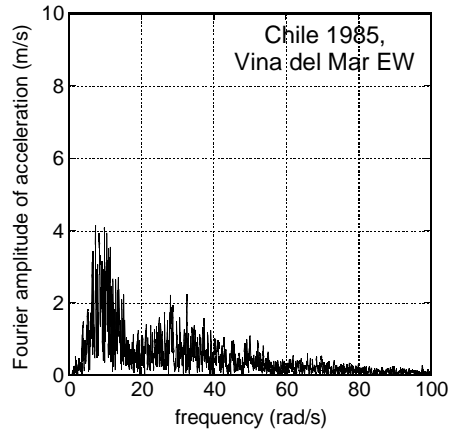
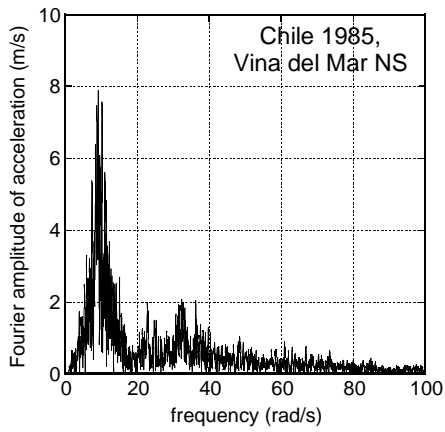


Fig.6(d) Long-duration soil motion

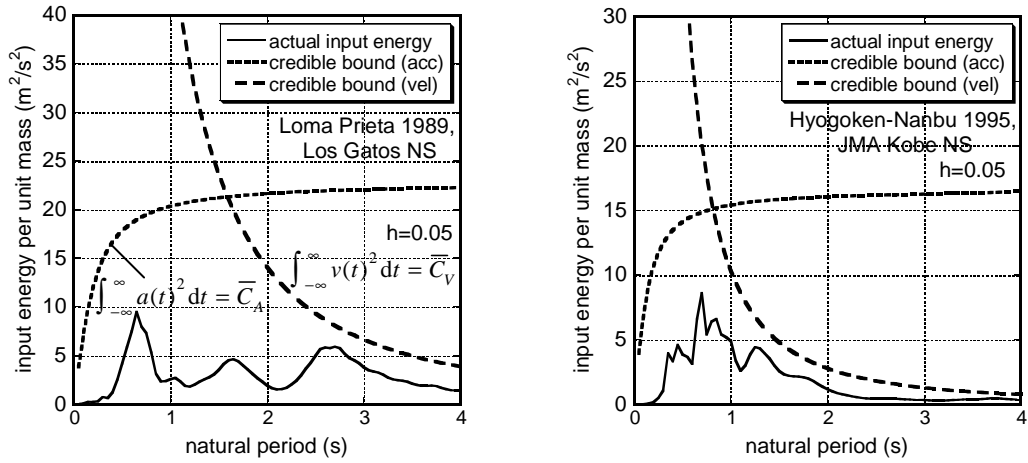


Fig.7(a) Near-fault rock motion

Fig.7 Actual earthquake input energy (damping ratio 0.05), credible bound for acceleration constraints and credible bound for velocity constraints:

- (a) Near-fault rock motion,
- (b) Near-fault soil motion,
- (c) Long-duration rock motion,
- (d) Long-duration soil motion

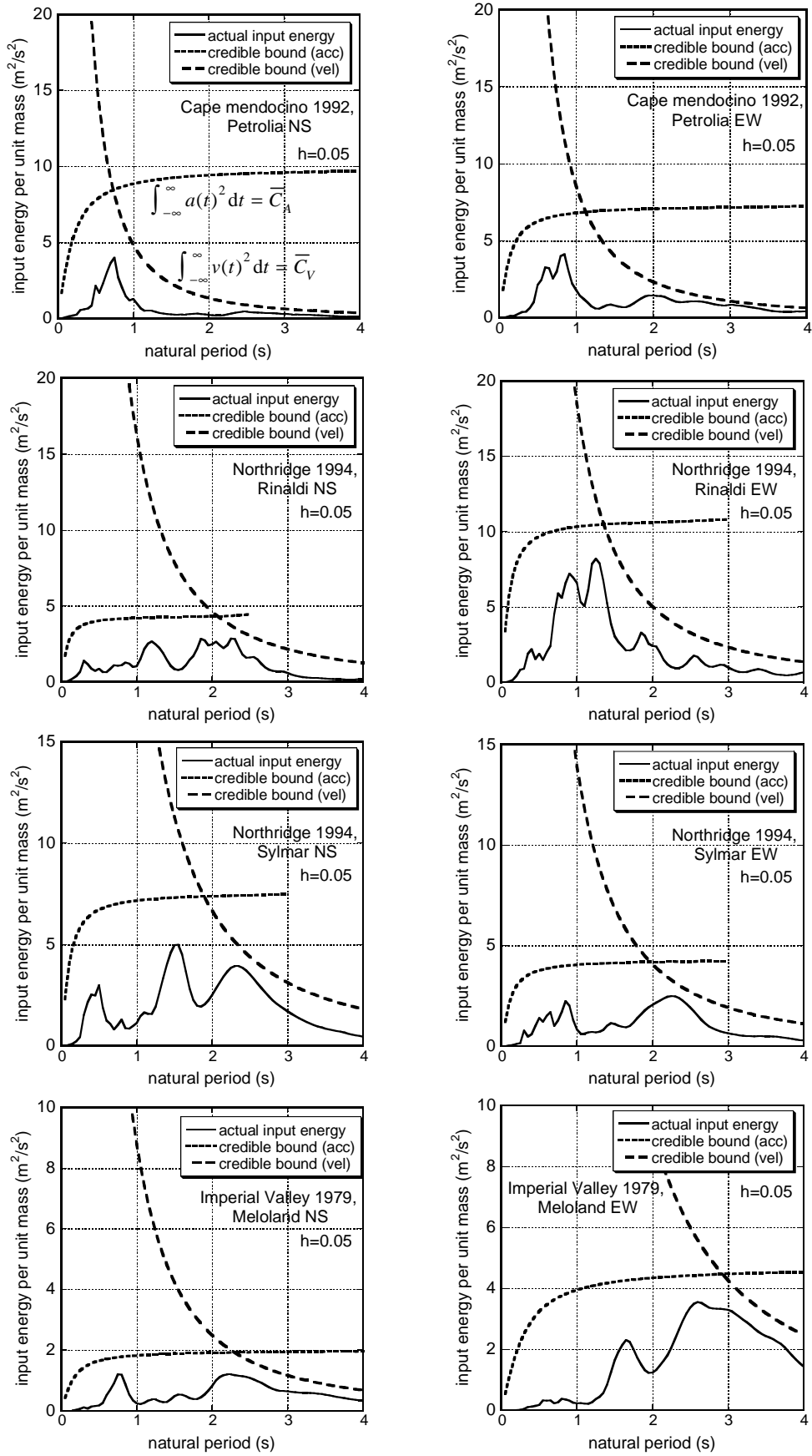


Fig.7(b) Near-fault soil motion

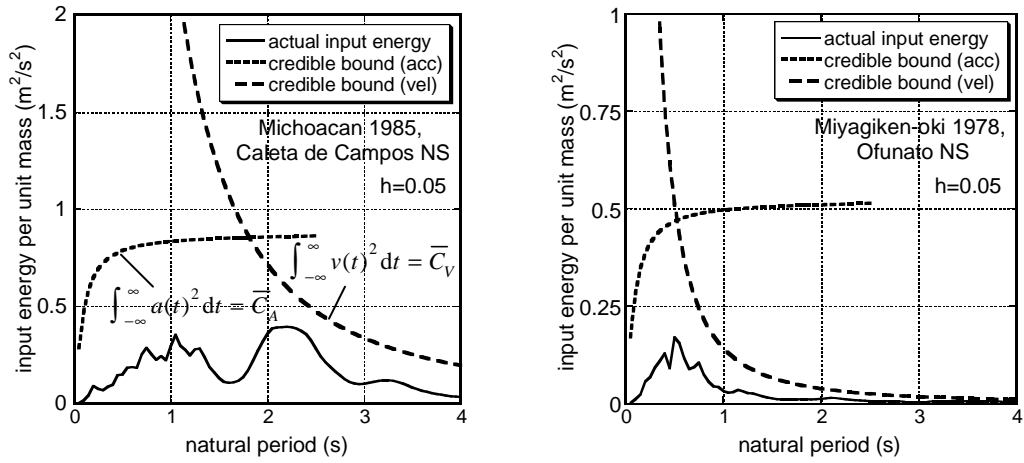


Fig.7(c) Long-duration rock motion

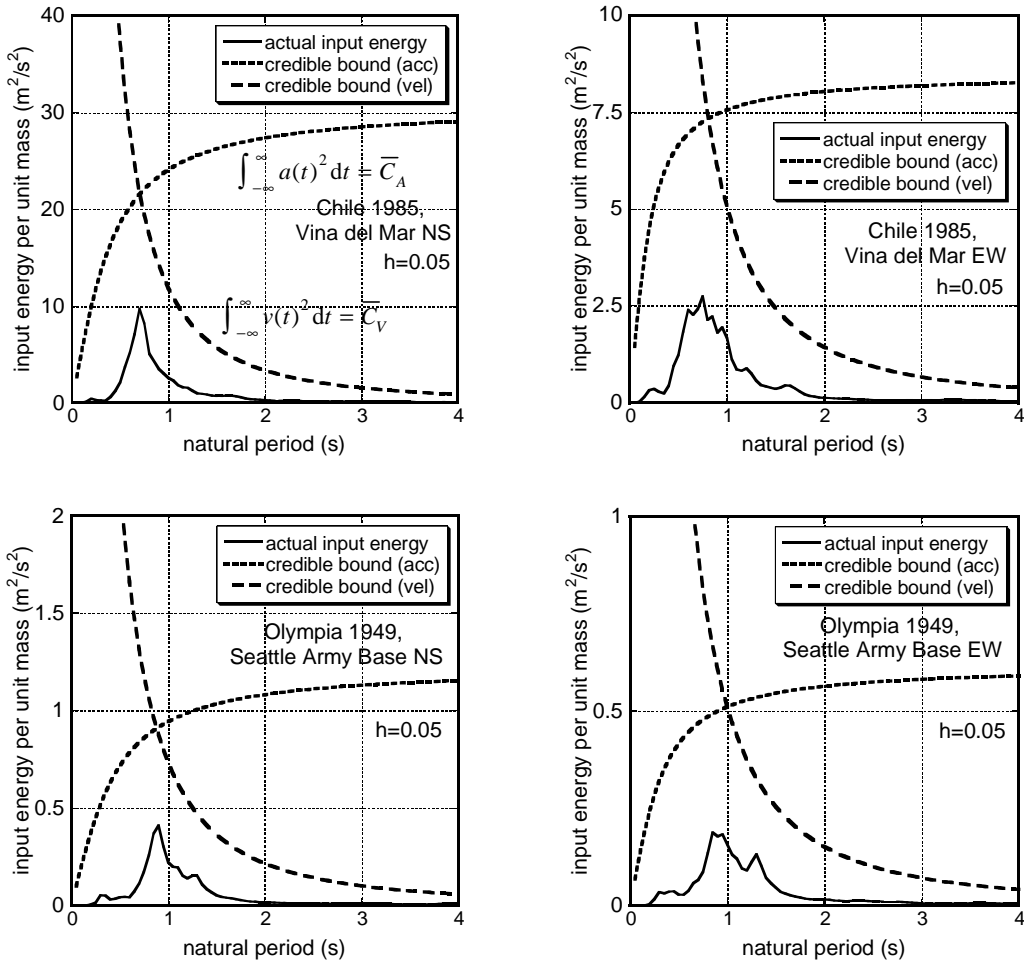


Fig.7(d) Long-duration soil motion

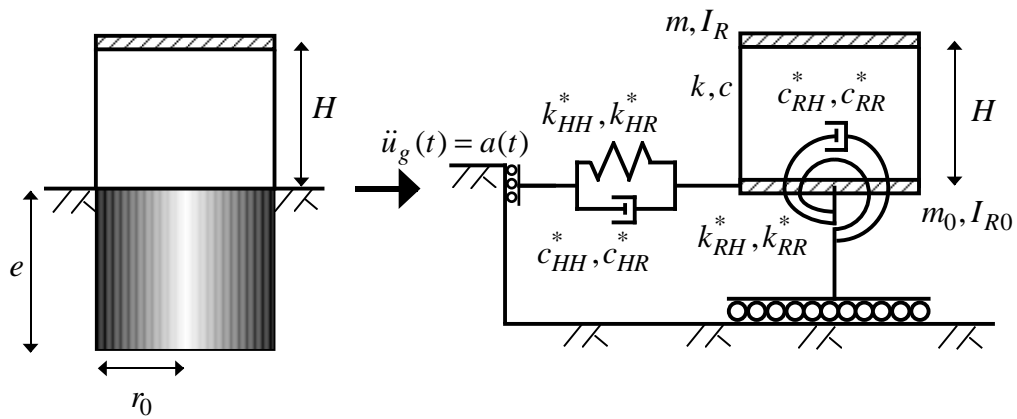


Fig.8 Linear elastic SDOF super-structure of story stiffness  $k$  and story damping coefficient  $c$  with a cylindrical rigid foundation embedded in the uniform half-space ground

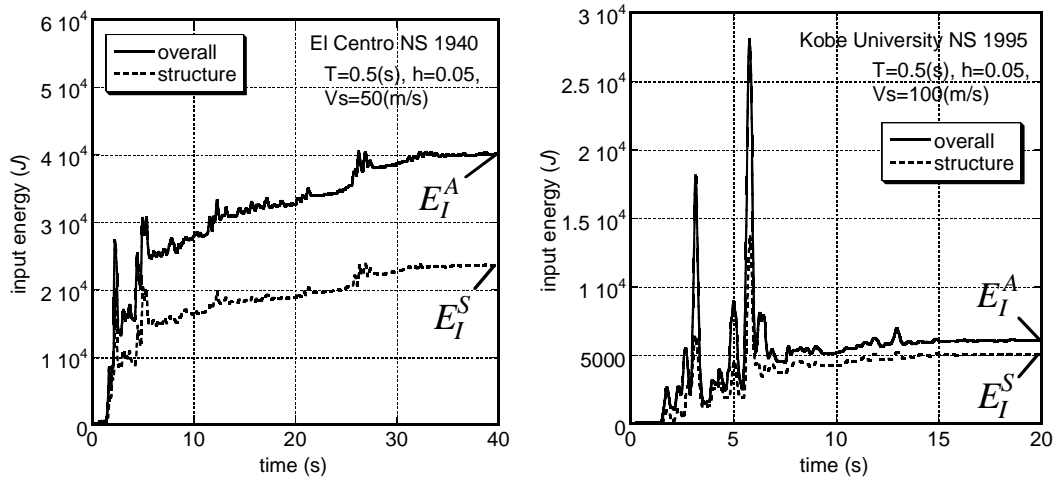


Fig.9 Examples of time histories of earthquake input energies (no embedment of foundation) and their final values  $E_I^A$ ,  $E_I^S$  for El Centro NS (Imperial Valley 1940) and Kobe University NS (Hyogoken-Nanbu 1995)

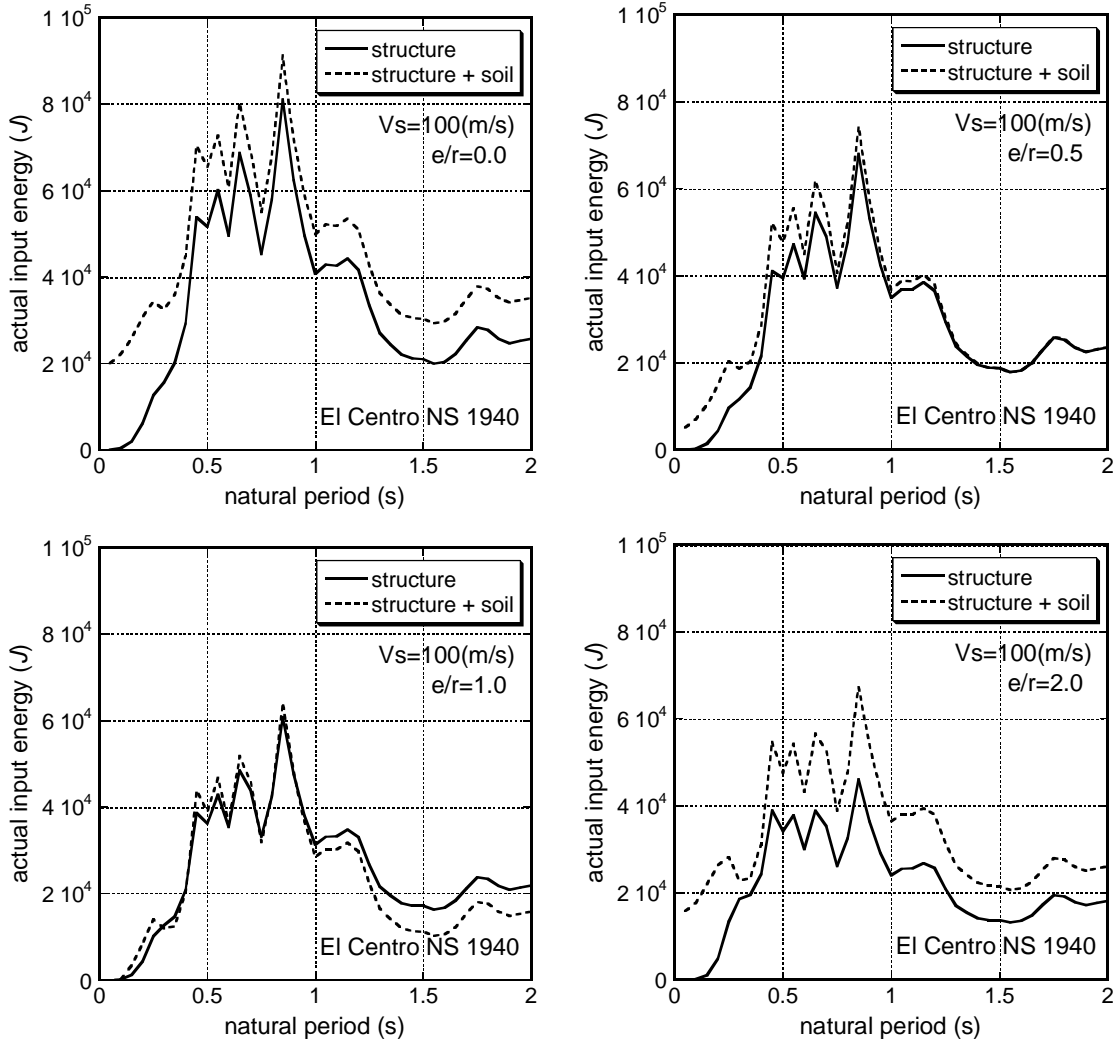


Fig.10 Earthquake input energies to the structure-soil system and the structure only with various degrees of foundation embedment for ground equivalent shear velocity 100 (m/s) to El Centro NS of Imperial Valley 1940

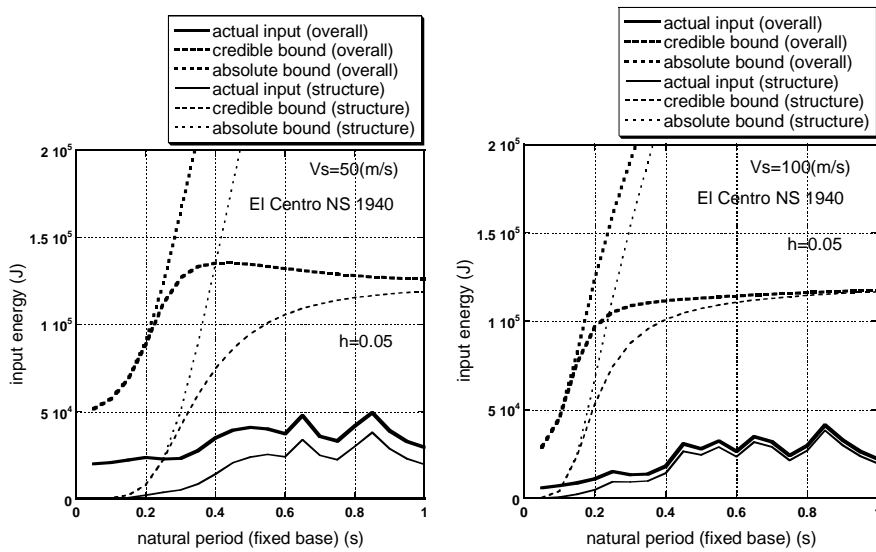


Fig.11: Earthquake input energies by the ground motion of El Centro NS 1940 to overall system and structure alone; (a)  $V_s=50$ (m/s), (b)  $V_s=100$ (m/s)

Table 1 Ground motions selected from PEER motions (Abrahamson et al. 1998)

earthquake	site and component	magnitude $M_w$	$\ddot{u}_{g \max}$ (g)	$\dot{u}_{g \max}$ (m/s)	$u_{g \max}$ (m)
<b>(Near fault motion/rock records)</b>					
Loma Prieta 1989	Los Gatos NS	6.9	0.570	0.988	0.379
Hyogoken-Nanbu 1995	JMA Kobe NS	6.9	0.833	0.920	0.206
<b>(Near fault motion/soil records)</b>					
Cape Mendocino 1992	Petrolia NS	7.0	0.589	0.461	0.265
	Petrolia EW		0.662	0.909	0.268
Northridge 1994	Rinaldi NS	6.7	0.480	0.795	0.505
	Rinaldi EW		0.841	1.726	0.487
	Sylmar NS		0.842	1.288	0.306
Imperial Valley 1979	Sylmar EW	6.5	0.604	0.778	0.203
	Meloland NS		0.317	0.711	1.242
	Meloland EW		0.297	0.943	3.124
<b>(Long duration motion /rock records)</b>					
Michoacan 1985	Caleta de Campos NS	8.1	0.141	0.255	1.464
Miyagiken-oki 1978	Ofunato NS	7.4	0.211	0.131	0.163
<b>(Long duration motion /soil records)</b>					
Chile 1985	Vina del Mar NS	8.0	0.362	0.337	2.400
	Vina del Mar EW		0.214	0.267	1.212
Olympia 1949	Seattle Army Base NS	6.5	0.0678	0.0785	0.192
	Seattle Army Base EW		0.0673	0.0777	0.0278

Table 2 Maximum Fourier amplitude spectrum of ground motion acceleration, time integral of squared ground motion acceleration and frequency band-width of critical rectangular Fourier amplitude spectrum of ground motion acceleration

earthquake	site and component	$A_{\max}$ (m/s)	$\bar{C}_A$ (m <sup>2</sup> /s <sup>3</sup> )	$\Delta\omega$ (rad/s)
<b>(Near fault motion/rock records)</b>				
Loma Prieta 1989	Los Gatos NS	6.80	49.5	3.36
Hyogoken-Nanbu 1995	JMA Kobe NS	5.81	52.3	4.87
<b>(Near fault motion/soil records)</b>				
Cape Mendocino 1992	Petrolia NS	4.49	21.5	3.35
	Petrolia EW	3.85	23.9	5.07
Northridge 1994	Rinaldi NS	2.98	25.0	8.84
	Rinaldi EW	4.70	46.3	6.58
	Sylmar NS	3.92	31.3	6.40
Imperial Valley 1979	Sylmar EW	2.95	16.3	5.88
	Meloland NS	2.01	5.43	4.22
	Meloland EW	3.09	6.93	2.28
<b>(Long duration motion /rock records)</b>				
Michoacan 1985	Caleta de Campos NS	1.33	3.97	7.05
Miyagiken-oki 1978	Ofunato NS	1.03	2.35	6.96
<b>(Long duration motion /soil records)</b>				
Chile 1985	Vina del Mar NS	7.87	34.3	1.74
	Vina del Mar EW	4.14	18.7	3.43
Olympia 1949	Seattle Army Base NS	1.57	1.28	1.63
	Seattle Army Base EW	1.12	0.877	2.20



Table 3 Maximum Fourier amplitude spectrum of ground motion velocity, time integral of squared ground motion velocity and frequency band-width of critical rectangular Fourier amplitude spectrum of ground motion velocity

earthquake	site and component	$V_{\max}$ (m)	$\bar{C}_V$ (m <sup>2</sup> /s)	$\Delta\omega$ (rad/s)
<b>(Near fault motion/rock records)</b>				
Loma Prieta 1989	Los Gatos NS	1.81	1.49	1.43
Hyogoken-Nanbu 1995	JMA Kobe NS	0.746	0.854	4.82
<b>(Near fault motion/soil records)</b>				
Cape Mendocino 1992	Petrolia NS	0.531	0.253	2.82
	Petrolia EW	0.697	0.509	3.29
Northridge 1994	Rinaldi NS	1.01	0.62	1.90
	Rinaldi EW	1.02	1.13	3.42
	Sylmar NS	1.22	0.858	1.81
	Sylmar EW	0.968	0.45	1.51
Imperial Valley 1979	Meloland NS	0.738	0.356	2.05
	Meloland EW	1.44	1.06	1.61
<b>(Long duration motion /rock records)</b>				
Michoacan 1985	Caleta de Campos NS	0.408	0.0759	1.44
Miyagiken-oki 1978	Ofunato NS	0.087	0.0119	4.89
<b>(Long duration motion /soil records)</b>				
Chile 1985	Vina del Mar NS	0.865	0.455	1.91
	Vina del Mar EW	0.563	0.199	1.97
Olympia 1949	Seattle Army Base NS	0.224	0.0232	1.45
	Seattle Army Base EW	0.189	0.0154	1.35